The Centenary of the Omori Formula for a Decay Law of Aftershock Activity

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8. *p* Values from Superposed Sequences

Why superposed Sequence?

- Many earthquakes followed by only a small number of recorded aftershocks
- Impossible to estimate p & c for each of these cases
- A set of superposed occurrence times measured from each main shock must fit the modified Omori formula at least approximately

Small p value from Superposed Sequences

- **Small p value for superposed sequences**
- The superposed sequences consists of mostly small-sized sequences (one or a few aftershocks)
- A portion of these may not be real aftershocks; **Only represent background seismicity ?**

Auther	Study region	Method	Main shock Mag.	time from Mainshock	p (omori formula)
Papazachos (1974)	Greece	superposed sequence of 2,544 aftershocks from 37 earthquakes (N >= 17)	M≥5.5 (for 1911-1965) M≥5.0 (for 1966-1972)		**1.13**
Davis & Frohlich (1991)	global	47,489 earthquakes were superposed. single-link cluster method for aftershock selection. This method links earthquakes occurring within 40 ST-km. (c; 0.03 day fixed)	M≥4.8 (ISC catalog)	0.1-20 day	**0.868**+_0.007
	･shallow subduction zone		M≥4.8 (ISC catalog)	0.1-20 day	**0.890**+_0.009
	･ridged-transform fault		M≥4.8 (ISC catalog)	0.1-20 day	**0.928**+_0.024
	･deep earthquakes (> 70 km)		M≥4.8 (ISC catalog)	0.1-20 day	**0.539**+_0.022
	shallow subduction zone	superposed aftershock sequence (N=1)	M≥4.8 (ISC catalog)	0.1-20 day	**0.777**
	shallow subduction zone	superposed aftershock sequence (N<=2)	M≥4.8 (ISC catalog)	0.1-20 day	**0.832**
	shallow subduction zone	superposed aftershock sequence (N<=5)	M≥4.8 (ISC catalog)	0.1-20 day	**0.831**
White & Reasenberg (1991)	Garm area of Tazihkistan (Peter- I fault zone)		?		**0.77**
	Garm area of Tazihkistan (north and south of the fault zone)		?		**1.0**
Ustu(1992)	Japan	superposed aftershock sequence (N>=1)	M≥4.0 (JMA catalog)	0.01-100 day	**0.914**(c=0.485)
	Japan	superposed aftershock sequence (N>=5)	M≥4.0 (JMA catalog)	0.01-100 day	**1.020**(c=0.107)
	Japan	superposed aftershock sequence (N>=20)	M≥4.0 (JMA catalog)	0.01-100 day	**1.070**(c=0.211)
Shaw(1933)	California	superposed sequence of 228 main shocks	3<=M<=6		**slightly less than 1**

* N: the number of aftershocks, * ST-km; space-time kilometers, assuming 1 day in time corresponds to 1 km in space

9. Anomalies in Aftershock Rate and Their Significance in Earthquake Prediction

Aftershock Anomalies in China

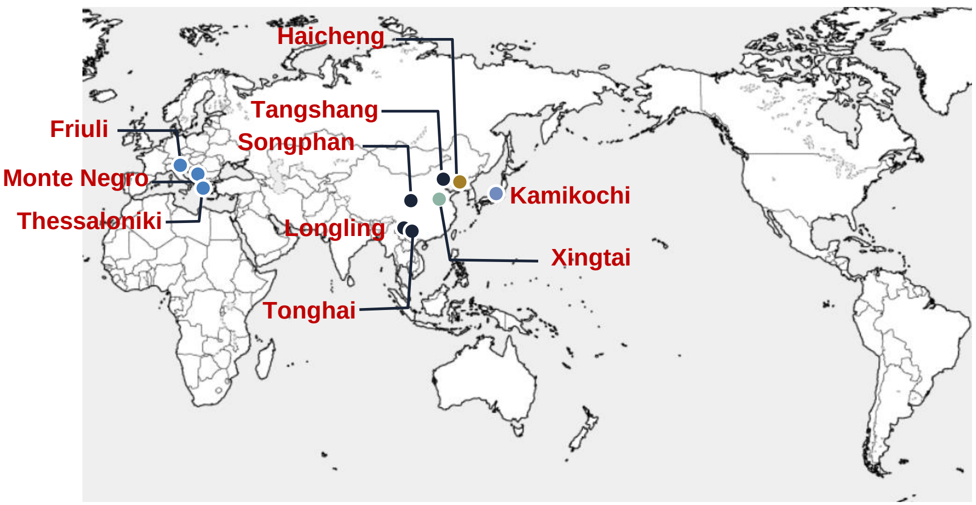
- **Aftershock activity** of 1975 Haicheng earthquake (M7.3) **decayed very rapidly** after an M6.0 earthquake on May 18, 1978 (Fu, 1981)
- **Relative increase in the rate of aftershocks** of the 1966 Xingtai earthquake 1~2 years prior to some large earthquakes (Wang, 1978, Li et al.,1980; Zhou et al., 1982)
- **When the decay of aftershock activity became slower, a large aftershocks followed**. This pattern was observed in seven large earthquakes of 7 large earthquakes of M > 7 in China since 1966 including the Longling, Tonghai, Songpan and Tangshang earthquakes (Xu, 1984)

Aftershock Anomalies in other region

- **Abnormally decreased aftershock activity followed by a large event** (Friuli in Italy ; Thessaloniki in Greece ; Monte Negro in Yugoslavia) (Schenkova et al., 1982)

Aftershock Anomalies in Japan

- Ohatake (1970) observed the aftershock sequence following a couple of earthquakes near Kamikochi, central Japan, on August 32 (M=4.7) and September 2 (M=5.0), 1969. **A remarkable decrease in aftershock activity began to 0.6 days after the former shock**.

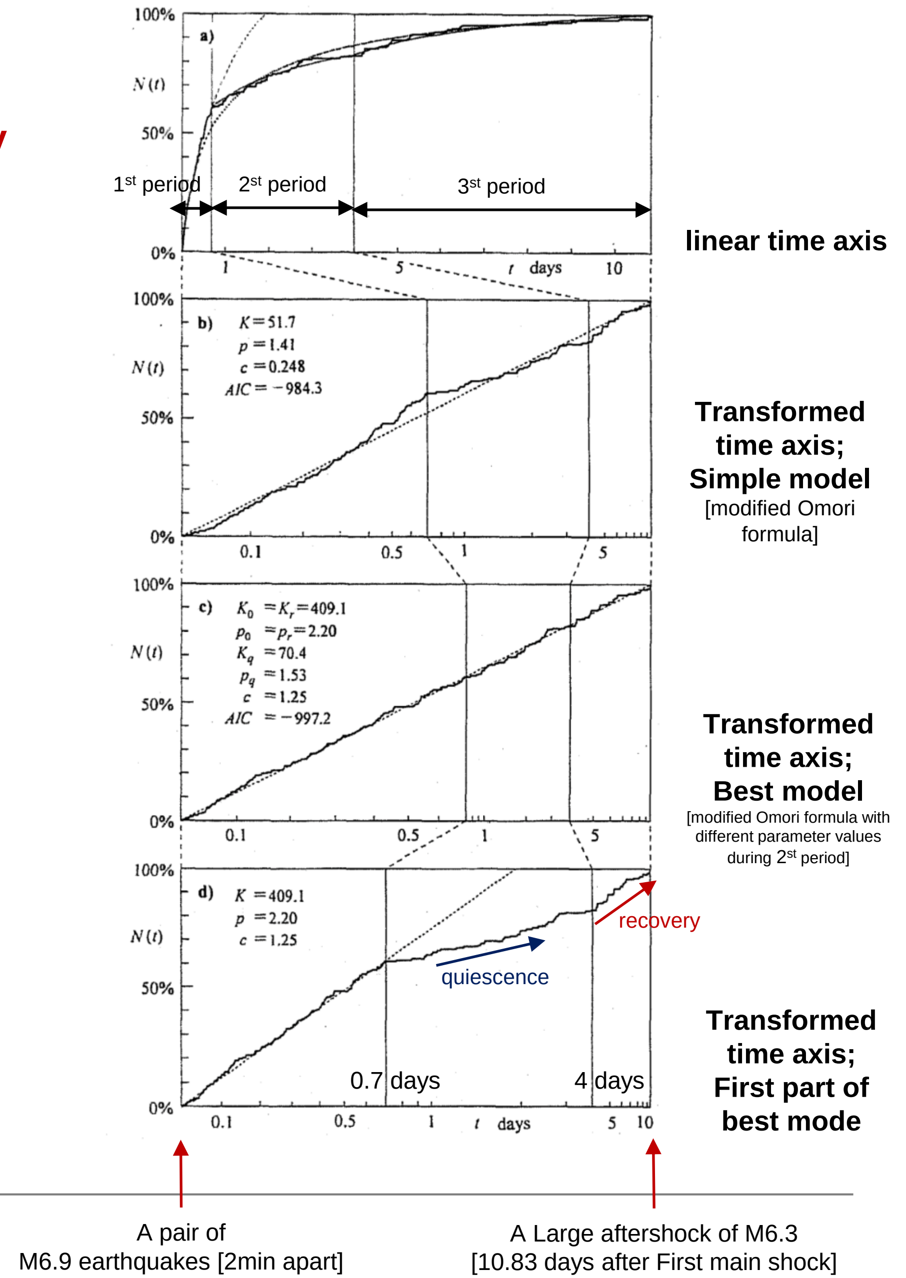


9. Anomalies in Aftershock Rate and Their Significance in Earthquake Prediction

Method to detect precursors statistically

- Matsu'ura (1986) closely investigated **the precursory decrease and recovery in aftershock activities before large aftershocks**.
  - **1984 Western Nagano earthquake** (Largest Aftershock M6.3 which occurred 22.5 h after Main Shock M6.8)
  - **1923 Kanto earthquake** (Large aftershock off Katsuura M7.3 which occurred 24 h after Main Shock M7.9)
  - **1992 Off the coast of Iwate Prefecture** (A pair of M6.9 earthquakes, 2min apart, occurred on July 18) **The quiescence & recovery are clearly seen in Fig.**

- Zhao et al. (1989) applied Matsu'ura's method to aftershocks of the Haicheng, Tangshan, Songpan, and Longling earthquakes in China and some of their foreshocks and obtained the similar results.



10. Duration of Aftershock Activity

Estimate of Background Seismicity of aftershock zone

■ Shiratori (1925) assumed the background seismicity was equal to **the average seismicity observed before the main shock**.

■ Watanabe (1989) compared the aftershock activity with **seismic activity of the surrounding region of the aftershock zone**.

Estimate of Duration using the level of background seismicity (μ)

- We fit a set of aftershock data to two models

$$n(t) = K(t+c)^{-p}$$

$$n(t) = K_2(t+c_2)^{-p_2} + \mu$$

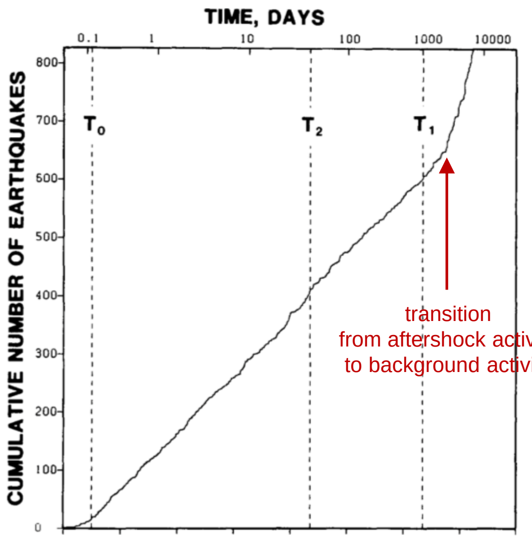
and compute Maximum likelihood estimates of the parameters & AIC values for each model.

- If ① $\mu > 0$ for the second model and ② AIC for the second model is smaller than the first, the duration t_d can be defined by

$$K_2(t_d + c_2)^{-p_2} = \mu$$

Application for natural earthquake

■ Ogata and Shimazaki (1984) estimated duration of aftershock activity accompanying the 1965 Aleutian earthquake (Mw=8.7).



- They fitted the aftershock data of M 4.7 from 3 h to 1,000 days to the modified Omori formula with a secondary sequence and estimated the parameters.

- **They concluded that a transition from aftershock activity to background activity occurred at this time.**

11. Other Related Studies

Dependence among aftershocks

- Jeffreys (1938), in his study of aftershocks of the 1927 Tango earthquake, concluded that **the aftershocks were mutually independent events.**

- Lomnitz (1966b) and Page (1968) found that there was clustering in small aftershocks, **but larger aftershocks were independent events.**

ETAS model vs Modified Omori formula

- In the ETAS model, **every aftershock may produce its own aftershocks.**

- The ETAS model usually provides **a smaller AIC than the modified Omori formula** for the same aftershock sequence.

- This indicates that **significant dependence exists among aftershocks.**

- However, the modified Omori formula remains to be a useful model for its simplicity.

12. Comparison with Other Decay Formulae

Modified Omori & Exponential Function

$$n(t) = Ke^{-\alpha t}(t + c)^{-p}$$

- Otsuka (1985, 1987) proposed a compound formula. For large t, the effect of the exponential function predominates.
- Exponential decay of activity in later periods of some aftershock sequences was suggested by Utsu (1957), Mogi (1962), Watanabe and Kuroiso (1970), and Otsuka (1985).
- Utsu (unpublished data, 1992) applied to several aftershock sequences and obtained maximum likelihood estimates of the parameters. The α values became zero, indicating that **this complication was unnecessary**.

Weibull distribution

$$f(t) = \alpha\beta t^{\beta-1} exp\left(-\alpha t^{\beta}\right)$$

- Some functions used in statistics show a decrease proportional to t^{-1} in a wide range of t, and a more rapid decrease for larger t
- Souriau et al. (1982) used the Weibull distribution to represent the time distribution of an aftershock sequence in the Pyrenees.
- A comparison of AIC between **the modified Omori formula** and Weibull distribution formula indicates that **the former fits better in most cases**

13. Application to Foreshock Sequences

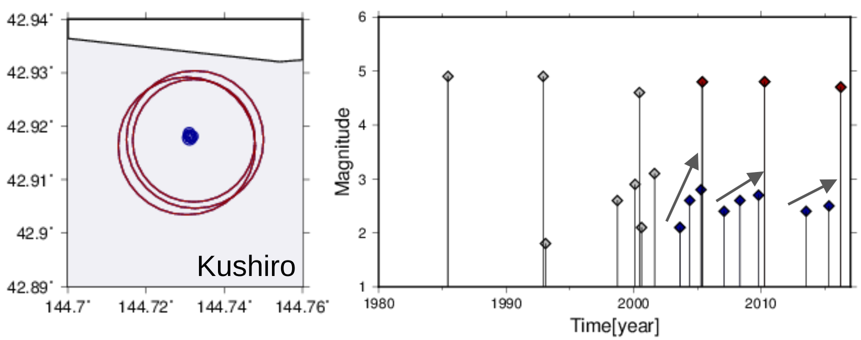
Three type of Foreshock sequences

- **Type 1, activity increases towards the main shock**
- Type 2, foreshock sequence itself has the form of a main shock-aftershock sequence or multiple occurrence of such sequences
- Type 3, irregular variation of activity like swarm

Type-1; activity increase toward Main- shock

- **Foreshock sequences of type 1 are rather rare** (e.g., Suzuki, 1985)

[Example type-1 (Okuda, Now researching)]



Reversed modified Omori formula

$$n(t) = K(t_0 - t)^{-p} \qquad (22)$$

- Papazachos (1973) proposed increasing function
- If the time is measured reversely from the main shock, **Eq. (22) takes the same form as the modified Omori formula**.
- Papazachos (1973) obtained high p values ranging from 1.53 to 2.60 for four foreshock sequences of dam-induced earthquakes

Moment release in a foreshock sequence

$$d\left(\sum\sqrt{M_0}\right)/dt = C\left(t_f - t\right)^{-n} \qquad (23)$$

Where $\sum\sqrt{M_0}$ represents the sum of the square roots of seismic moment (or energy) released until t, and C, t_f and n are constants to be estimated from the observational data. The main shock is expected to occur at $t = t_f$.

- Varens (1989) proposed an equation for seismic moment release in a foreshock sequence
- **Similar methods have been used in predicting volcanic eruptions for more than 30 years by several volcanologists.**

14. Point Process Models Incorporation the Modified Omori Formula

The trigger model

$$g(t) = Af(t - t_f) \quad for \quad t \geq t_f$$
$$g(t) = 0 \quad for \quad t < t_f$$

- Neyman-Scott type model proposed by Vere-Jones and Davies (1966) in a study of seismicity of New Zealand
- In this model, the seismicity consists of two kinds of events, primary and secondary
- Primary events (main shocks) distribute as a Poisson process with constant occurrence rate . Each primary event occurring at time triggers a series of secondary events (aftershocks)

Application for natural earthquake

- Modified Omori type function for $f(t)$ (Vere-Jones & Davies, 1966 ; Utsu, 1972; Sase, 1974)
- Exponential type function for $f(t)$ (Vere-Jones & Davies, 1966)
- Delta function $\delta(t)$ for $f(t)$ (Shlien and Toksoz, 1970)

14. Point Process Models Incorporation the Modified Omori Formula

The ETAS model FORMULA

$$\lambda(t) = \mu + \sum_{t_i<t} g\,(t - t_i) \tag{26}$$

$$g(t - t_i) = K \exp\{\alpha(M_i - M_z)\}\,(t - t_i + c)^{-p} \tag{27}$$

$$\Lambda(t) = \int_{T_s}^{t} \lambda(s)ds = \mu(t - T_s) + K \sum_{t_i<t} \exp\{\alpha(M_i - M_z)\}\,\{c^{1-p} - (t - t_i + c)^{1-p}\}/(p - 1) \tag{28}$$

$\lambda(t)$;The seismic activity. μ ;background activity.
$g(t - t_i)$; the rate of activity at time t triggered by an event at time t_i

K, a, c, and p are constants common to all aftershock sequences. $\Lambda(t)$The cumulative number of earthquakes at time t. μ, K, c, a, and p represent characteristics of seismic activity of the region.

The ETAS model

- The ETAS model (Ogata, 1986, 1988, 1989, 1992, 1994) is a self-exciting point process model.
- **Most p and c values obtained for various earthquake data sets fall in the range between 0.9 and 1.4, and between 0.003 and 0.3 days, respectively** (Ogata, 1986, 1988, 1992).
- **The parameter α, mostly ranging from 0.2 to 3.0, measures an efficiency** of a shock with a certain magnitude in generating its aftershock activity.

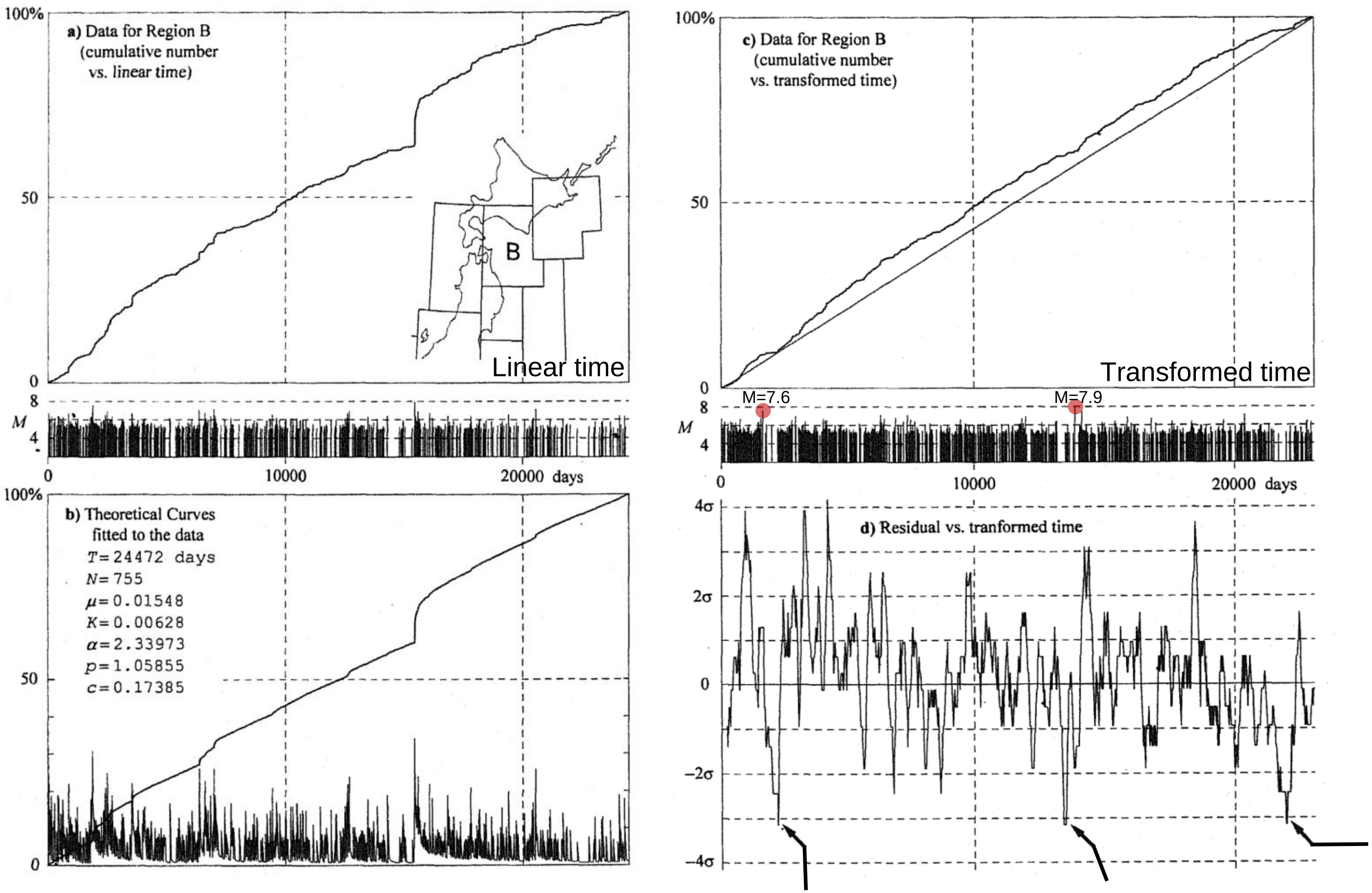
- Swarm-type activity ; Small α
- Main shock-aftershock activity ; Large α

Application for natural earthquake

- Utsu (unpublished data, 1993) divided Japan into 16 regions and applied the ETAS model to shallow earthquakes (depth < 100km) in each region during the years 1926-1992.
- To keep temporal homogeneity of data, the lowest limit of magnitude Mz was chosen as 4.45, 4.95, or 5.45 depending on the region.
- The p and c values for the 16 regions range from 0.95 to 1.22 and from 0.003 to 0.28 days, respectively

14. Point Process Models Incorporation the Modified Omori Formula

ETAS model applied to shallow earthquakes vs. time diagram



M>5.0; 1926-1992
depth < 100km

1st ; Deficiency of aftershocks of the 1931 earthquake (M7.6)

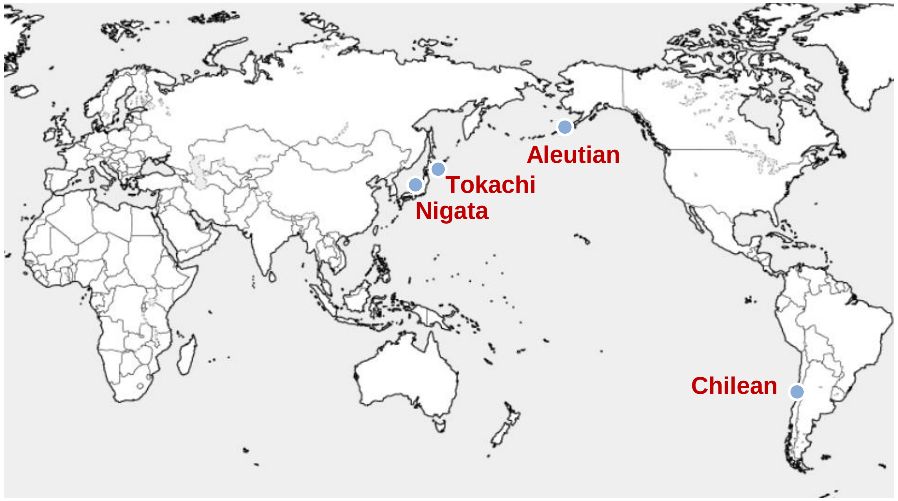
2nd **precursory quiescence ?** (before the 1968 Tokachi-Oki earthquake; M=7.9)

3rd **Related to a large earthquake swarm ?** (in the fall of 1989 near the southeastern border of this region, including M7.1)

14. Point Process Models Incorporation the Modified Omori Formula

Residual analysis from the ETAS model

- 1964 Niigata and 1952 Tokachi-Oki earthquakes. (Inoue, 1965)
- 1960 Chilean, 1957, 1965 and 1986 Aleutian, and 1968 Tokachi-Oki earthquakes (Ogata, 1992)



15. Conclusion

- The decay of aftershock activity with time is generally represented by the modified Omori formula with the exponent **p usually between 0.9 and 1.5.**

- This unique nature of aftershock activity provides a strong constraint in constructing a physical theory of aftershock generation.

- In point process modeling of shallow seismicity in the time domain, the effect of the aftershock activity, most suitably represented by the modified Omori formula, must be considered.

- At present, the ETAS model seems to be useful for representing the general seismicity of a region with relatively few parameters.

- **Parameters of the modified Omori formula and the ETAS model may correlate with tectonophysical conditions** (structural heterogeneity, stress state, temperature, etc.), therefore they may vary spatially and in some cases temporally.

- **Decrease of the observed activity from the level predicted by the modified Omori formula or the ETAS model may be followed by a large earthquake**.