Seismogenesis Seminar 2017S

Mainshocks are aftershocks of conditional foreshocks: How do foreshock statistical properties emerge from aftershock laws

Helmstetter et al. (2003)

2017/06/05

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Abstract

Main topic:

Using the ETAS model, authors derived the inverse Omori law $N(t) \sim 1/(t_c - t)^{p'}$ for foreshocks from the Omori law $N(t) \sim 1/(t - t_c)^p$ for aftershocks.

They also verified the decrease of the b-value of the GR law before the mainshock and the inward migration of foreshocks toward the mainshock.

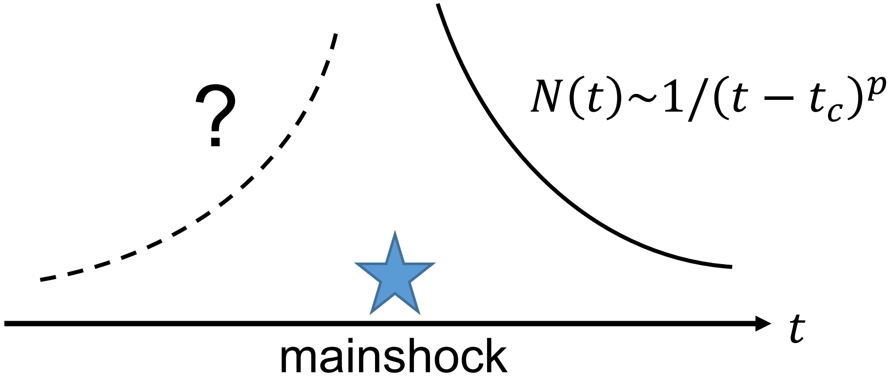
1. Introduction

Foreshock and Aftershock

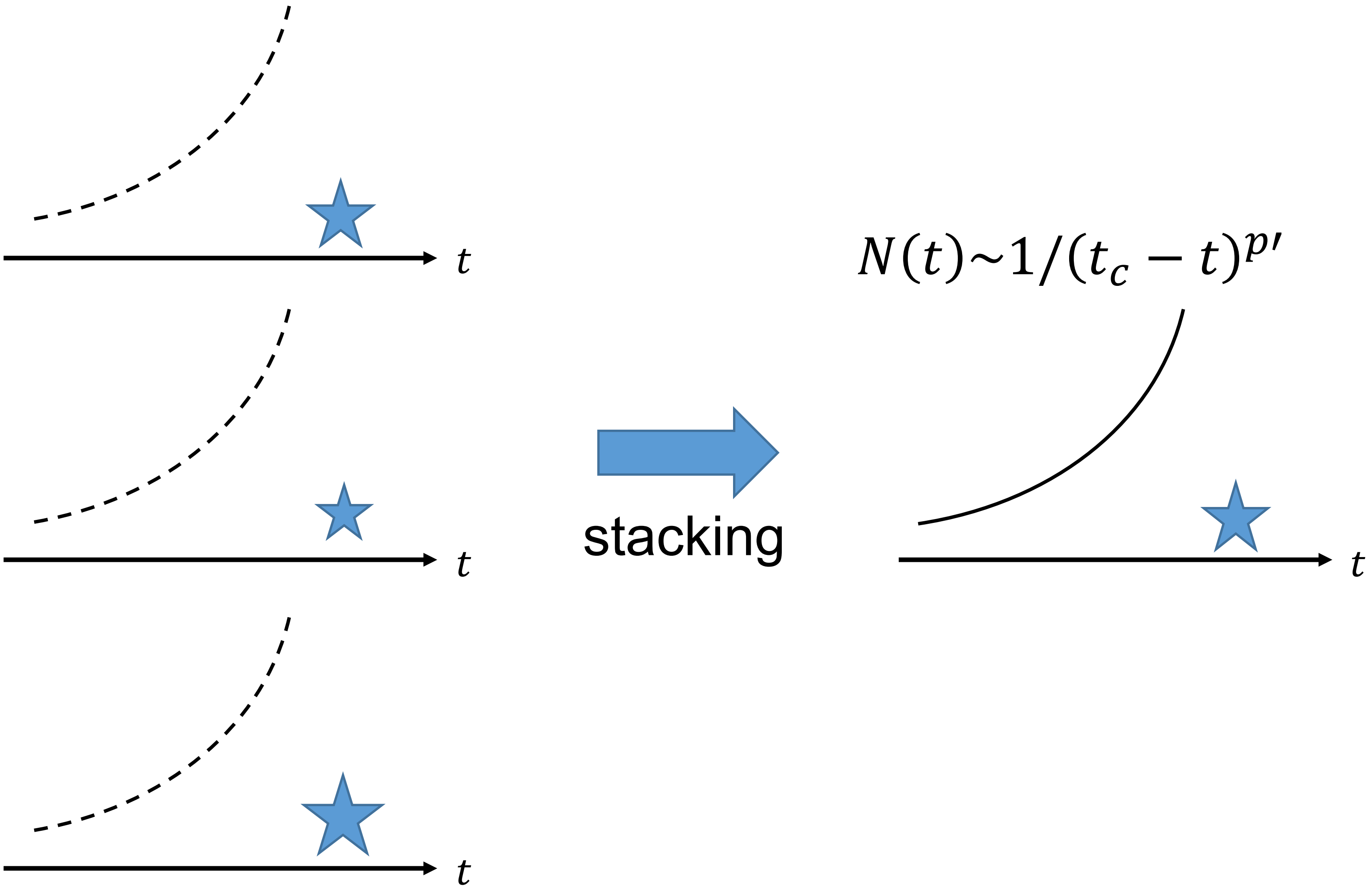
For an aftershock sequence, Omori law $N(t) \sim 1/(t-t_c)^p$ has been proposed [Omori, 1894] and has since been verified by many studies.

For foreshocks, there were not such a well-defined empirical law because of the fluctuation of the foreshock seismicity rate and the small number of foreshocks.

It is impossible to establish a deterministic law when looking at a single foreshock sequence.



By synchronizing foreshock sequences at the time of their mainshocks and stacking the sequences, the seismicity follows an inverse Omori law $N(t)\sim 1/(t_c - t)^{p'}$ (1970s).



stacking

$N(t)\sim 1/(t_c - t)^{p'}$

In this study,
using only the physics of aftershocks (GR and Omori laws), authors aims to derive the observed properties of foreshock.

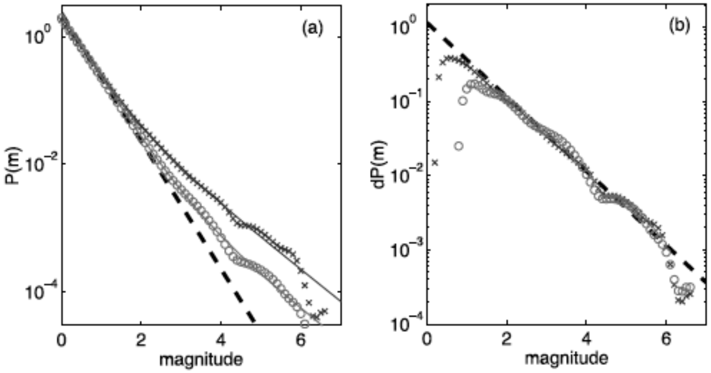
Tool: ETAS model

Main Results:

1. The inverse Omori law $N(t) \sim 1/(t_c - t)^{p'}$

2. The foreshock energy distribution changes before the mainshock. This may explain the low b-value before large earthquakes.

3. Modification of the GR distribution for foreshocks is shown analytically and verified by numerical simulations.



4. Foreshock migration toward the mainshock.

Flow

2. Definition of the ETAS model: formulation

3. Derivation of the inverse Omori law

4. Prediction for the GR distribution of foreshocks

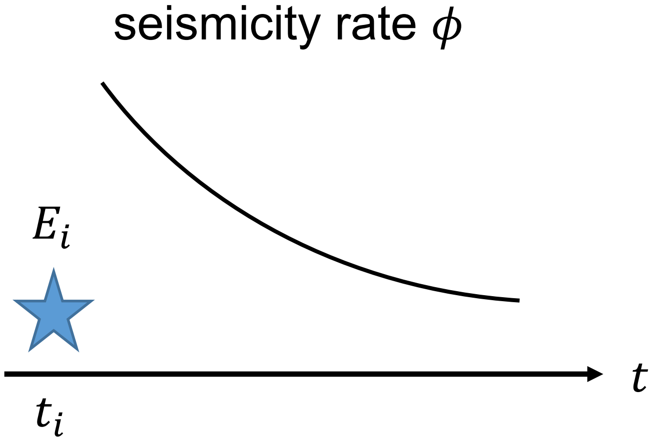
5. Migration of foreshocks toward the mainshock

6. Discussion
7. Conclusion

2. Definition of the ETAS model

2.1 Definition

$$\boxed{\phi_{E_i}(t-t_i)=\rho(E_i)\Psi(t-t_i)}$$



average number of daughters born from a mother of E_i

$$\rho(E_i)=k\left(\frac{E_i}{E_0}\right)^a$$

density distribution about time (bare Omori law)

$$\Psi(t-t_i)=\frac{\theta c^\theta}{(t-t_i+c)^{1+\theta}}$$

The energy distribution obeys the GR law

$$P(E)=\frac{\beta E_o^\beta}{E^{1+\beta}}$$

The total seismicity rate $\lambda(t)=s(t)+\sum_{i|t_i<t}\phi_{E_i}(t-t_i)$

($s(t)$ is the background seismicity)

2.2 The Master equation

Taking the expectation (ensemble average), we obtain the Master equation for the statistical average $N(t)$

$$N(t) = \mu + \int_{-\infty}^{t} d\tau \phi(t-\tau) N(\tau),$$

here μ is the expectation of the stationary background and $\phi(t) = \int_{E_0}^{\infty} dE' P(E') \phi_{E'}(t)$.

To solve the equation, they introduce the Green Function $K(t)$

$$K(t) = \boxed{\delta(t)} + \int_{0}^{t} d\tau \phi(t-\tau) K(\tau).$$

Physically, $K(t)$ is the renormalized Omori law including secondary, tertiary, …, events.

↔ $\Psi(t) \sim 1/t^{1+\theta}$: the bare Omori law

The solution of the green function $K(t)$:

$K(t) \sim 1/t^{1-\theta}$ for $\mathrm{c} < \mathrm{t} < t^{*}$,

$K(t) \sim 1/t^{1+\theta}$ for $\mathrm{t}^{*} < t$,

where $\mathrm{t}^{*} \approx c(1-n)^{-1/\theta}$.

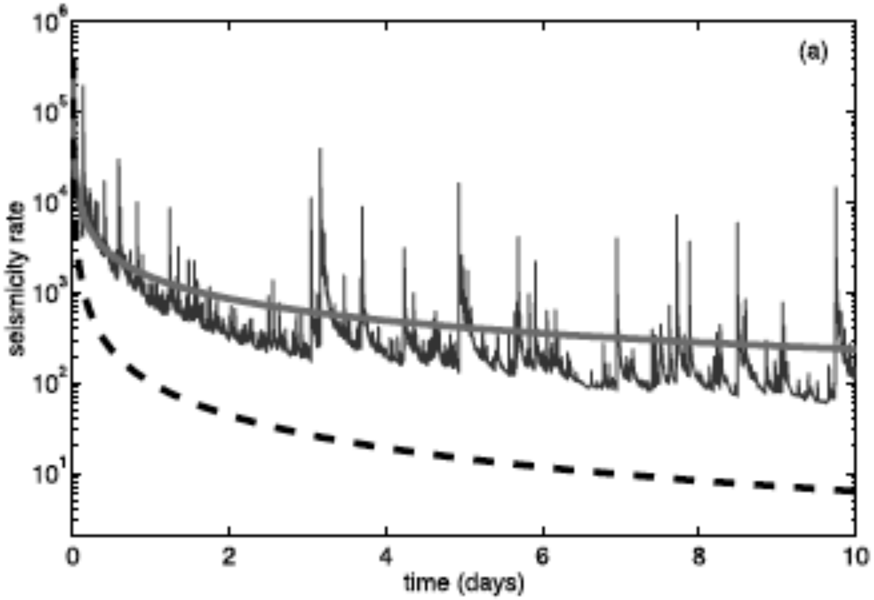
n is the average number of daughters created per mother event

$$n = \int_{E_0}^{\infty} dEP(E)\rho(E).$$

GR law

average number of daughters created by an event of *E*

The result of a numerical simulation



noisy solid line: observed seismicity rate

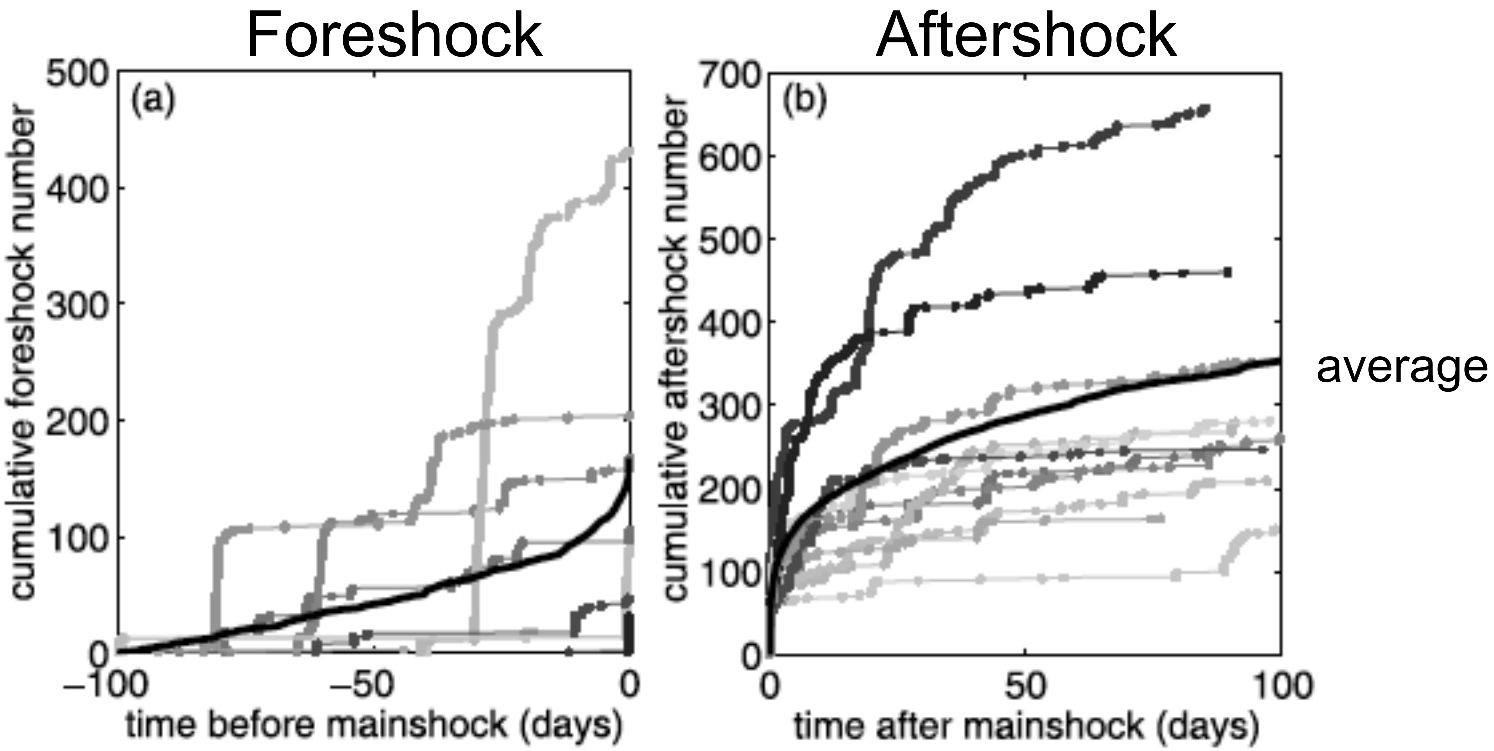
solid line: the renormalized Omori law

$$K(t) \sim 1/t^{1-\theta}$$

dashed line: the bare Omori law

$$\Psi(t) \sim 1/t^{1+\theta}$$

3. Derivation of the inverse Omori law



11 sequences of foreshocks and aftershocks generated by the ETAS model.

The Omori law for the aftershocks is clearly observed for any sequence

The inverse Omori law for the foreshocks can be seen only after the averaging.

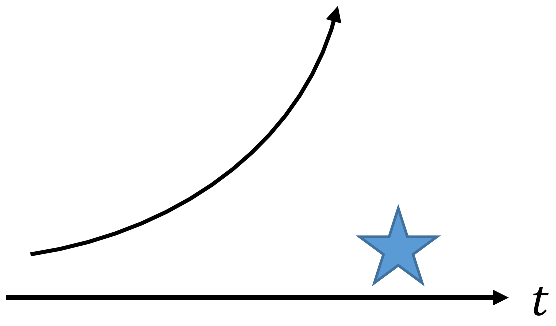
The inverse Omori law is a statistical statement.

3. Derivation of the inverse Omori law

We assume the subcritical regime $n < 1$.

n is the average number of daughters created per mother event, so $n < 1$ means that the seismicity rate decays.

How is it possible in this framework to get an accelerating seismicity preceding a large event?



The answer is that we see only the sequences of events which ends at a mainshock.

The existence of a mainshock requires that a specific sequence of noise realization must have taken place to ensure its existence.

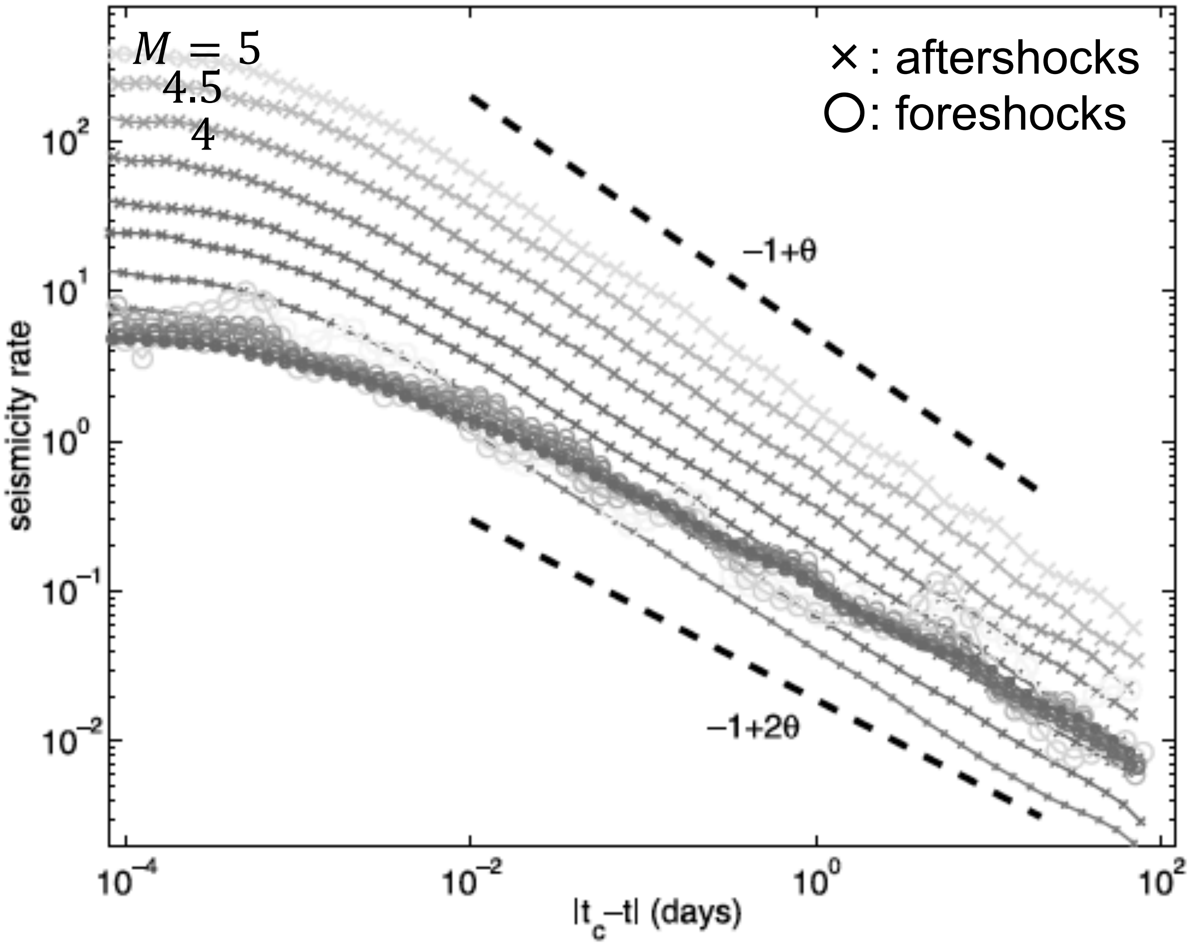
The conditioning of the existence of a mainshock influences the seismicity prior to the mainshock.
This conditioning is expressed as $\lambda(t=0)=\langle N\rangle+\lambda_0$.
(λ_0: burst of seismic activity)

Then, the expectation of seismicity rate before the mainshock is

$$E[\lambda(t)|\lambda_0] \propto \frac{\lambda_0}{(t_c-t)^{1-2\theta}} \qquad \left(a<\frac{\beta}{2}\right)$$

or

$$E[\lambda(t)|\lambda_0] \propto \frac{\lambda_0}{(t_c-t)^{1-\frac{\beta}{a}\theta}} \qquad \left(a\geq\frac{\beta}{2}\right)$$



For aftershocks, the time decay exponent

$p = 1 - \theta \quad \left(K(t) \sim \frac{1}{t^{1-\theta}}\right).$

For foreshocks, the time decay exponent

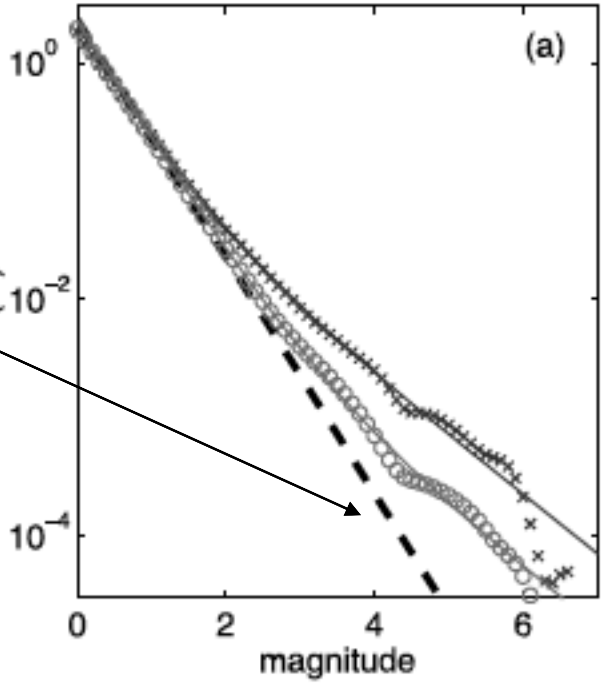
$p' = 1 - 2\theta \quad \left(E[\lambda(t)|\lambda_0] \propto \frac{\lambda_0}{(t_c - t)^{1-2\theta}}\right).$

4. Prediction for the GR distribution of foreshocks

Similar to the case of the time dependency (Inverse Omori law), the conditional GR distribution for foreshocks can be derived as follows:

$$P[E|\lambda_0] \sim \frac{E_0^{\beta}}{E^{1+\beta}} + \frac{C}{(t_c - t)^{\theta(\beta-a)/a}} \frac{E_0^{\beta'}}{E^{1+\beta'}} \qquad (\beta' = \beta - a)$$

normal GR distribution

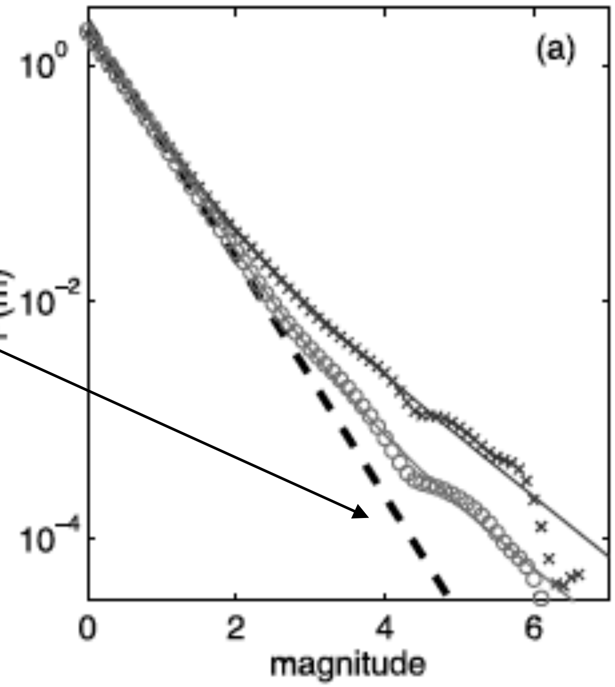


×: $t_c - t < 0.1$ day

○: 1 day $< t_c - t < 10$ days

solid line: prediction

normal GR distribution



×: $t_c - t < 0.1$ day

○: 1 day $< t_c - t < 10$ days

solid line: prediction

The prediction line seems to have the decreased b-value, but more accurately, this is due to the additive term $\propto \frac{1}{E^{1+\beta\prime}}$.

$$P[E|\lambda_0]{\sim}\frac{E_0^{\beta}}{E^{1+\beta}}+\frac{C}{(t_c-t)^{\theta(\beta-a)/a}}\frac{E_0^{\beta\prime}}{E^{1+\beta\prime}} \qquad (\beta' = \beta - a)$$

5. Migration of foreshocks toward the mainshock

Similar to the cases of the time dependency (Inverse Omori law) and the magnitude (conditional GR distribution), the seismic rate $\lambda(\boldsymbol{r},t)$ conditioned on the existence of the mainshock can be derived as follows:

$$E[\lambda(\boldsymbol{r},t)|\lambda(\boldsymbol{0},t_c)]\sim \int_{-\infty}^{t} d\tau \int d\rho K(\boldsymbol{r}-\boldsymbol{\rho},t-\tau)K(\boldsymbol{\rho},t_c-\tau)$$

If the probability distribution for an earthquake to trigger an aftershock at a distance *r* is of the form

$$\rho(r)\sim\frac{1}{(r+d)^{1+\mu}},$$

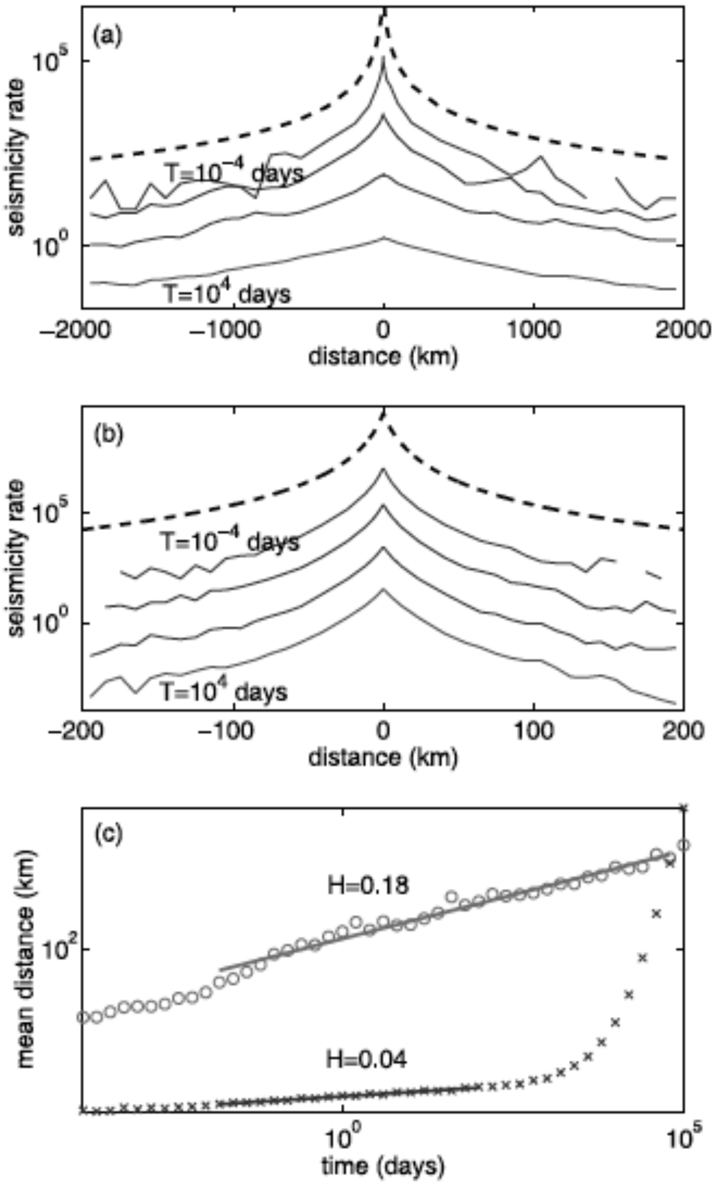
the characteristic distance of aftershocks *R* diffuses as $R\sim t^H$,
$H=\frac{\theta}{\mu}$ for $\mu<2$, $H=\frac{\theta}{2}$ for $\mu>2$.
This result is valid for foreshocks.

Foreshock migration: $R \sim t^{H}$
The shorter the time period *T* before the mainshock, the nearer to the mainshock the events happen.

(a) $H = 0.2$

(b) $H = 0.01$

Panel (c) shows the characteristic distance as a function of the time to the mainshock.
The observed inclination *H* well agree with the predictions (solid line).



6. Discussion

6.1 Difference between type I and type II foreshocks

How to define foreshocks.

type I:

in a certain space-time window $R{\times}T$, foreshocks are the events of magnitude smaller than or equal to the following events.

$$M \leq M_{\mathrm{main}}$$

type II:

foreshocks are the events preceding a large earthquake.

$$t \leq t_c$$

In this study, the authors used the type II. However, their results remain robust if using the type I definition.

6.2 Inverse Omori law

Their results about the inverse Omori law reproduced both the variability of p’ and the lower value measured for p’ than for p.

Their results are statistical, so fundamentally different from the critical point theory [Sammis and Sornette, 2002] which leads to a power law increase of seismic activity preceding each single large earthquake.

6.3 Foreshock occurrence rate

Large mainshocks have significantly more aftershocks than foreshocks, while small EQs have roughly the same number of foreshocks and of aftershocks.

The ratio $\frac{\#\text{ of aftershock}}{\#\text{ of foreshock}}$ increases if the ratio $\frac{a}{\beta}$ decreases.

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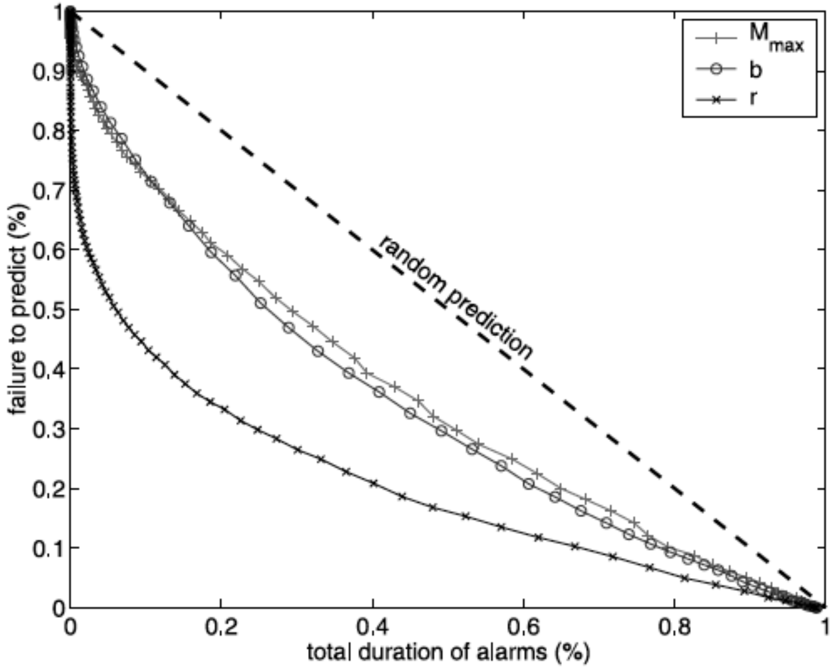
6.5 Implications for earthquake prediction

The inverse Omori law and the apparent decrease of *b*-value have been derived as statistical laws conditioned on leading to a burst of seismicity at the time of the mainshock.

This does not mean that there is not a physical content in these laws.



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