

Rupture dynamics with energy loss outside the slip zone

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[1] Energy loss in a fault damage zone, outside the slip zone, contributes to the fracture energy that determines rupture velocity of an earthquake. A nonelastic two-dimensional dynamic calculation is done in which the slip zone is modeled as a fault plane and material off the fault is subject to a Coulomb yield condition. In a mode 2 crack-like solution in which an abrupt uniform drop of shear traction on the fault spreads from a point, Coulomb yielding occurs on the extensional side of the fault. Plastic strain is distributed with uniform magnitude along the fault, and it has a thickness normal to the fault proportional to propagation distance. Energy loss off the fault is also proportional to propagation distance, and it can become much larger than energy loss on the fault specified by the fault constitutive relation. The slip velocity function could be produced in an equivalent elastic problem by a slip-weakening friction law with breakdown slip D_c increasing with distance. Fracture energy G and equivalent D_c will be different in ruptures with different initiation points and stress drops, so they are not constitutive properties; they are determined by the dynamic solution that arrives at a particular point. Peak slip velocity is, however, a property of a fault location. Nonelastic response can be mimicked by imposing a limit on slip velocity on a fault in an elastic medium.

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1. Introduction

[2] This paper is concerned with energy loss near a dynamically propagating rupture front in an earthquake. *Sibson* [2003] describes a fault zone as consisting of a relatively thin principal slip zone surrounded by a damage zone (cataclasite zone). In a dynamic event on a preexisting fault, friction changes from its static value to a sliding value in a zone variously termed the breakdown zone, the process zone, or the cohesive zone. High stress associated with the breakdown zone is not confined to the fault plane, but exists in a volume around the rupture front. Similar terminology is applied to the transition from intact rock to sliding on a fracture surface. In that context, *Scholz et al.* [1993, p. 21,956] state that “The stresses within the process zone will be much higher than the ambient tectonic stresses and are likely to be the highest stresses the rock has experienced. Thus the passage of the fault tip will leave behind it a wake of damaged rock.” In the context of earthquakes on an existing fault, *Scholz et al.* [1993, p. 21,958] state that “the cataclasite zone may be the result of pulverization due to repeated passage of the crack tip stress field from many earthquakes.”

[3] Linear fracture mechanics predicts that, for a crack with uniform stress drop, the slip distribution is elliptical with vertical slope at the tips. That prediction is altered

within a breakdown zone, so that slip tapers to zero gradually. *Cowie and Scholz* [1992] and *Scholz et al.* [1993] examined field data of slip on discrete faults with finite length and found that the slip distribution is tapered and is self-similar. Making the assumption that the geologic slip distribution is similar to that in a single event, they conclude that the size of the breakdown zone scales with fault length, which implies that fracture energy G , the energy loss per unit fault area, is proportional to fault length L .

[4] *McGarr et al.* [2004] observe that laboratory values of fracture energy G can be reconciled with values inferred for earthquakes if the value of G appropriate to an event is proportional to maximum slip in the event.

[5] *Moore and Lockner* [1995] examined a laboratory sample, deformed by differential stress with all principal stress components compressive, in which a developing shear fracture had been arrested. They found increased microcrack density in a volume surrounding the fracture tip. Microcracks that are evident are tensile cracks, with lengths ranging up to the grain size. They presumably formed in association with slip on grain boundaries, which would be less evident in the photomicrographs. *Vermilye and Scholz* [1998] examined thin sections taken from the vicinity of several small natural faults in brittle rock. They found tensile microcracks at two different orientations, which they interpreted as arising from the passage of mode 2 crack tips in opposite directions. The density of microcracks was constant along the fault trace and decreased

normal to the fault with a width at half maximum on the order of 10^{-2} times fault length.

[6] *Yamashita* [2000] performed a finite difference calculation of a mode 2 rupture in which tensile microcracks formed when stress reached absolute tensile values. Microcracks occurred only on the extensional side of the fault. Microcrack density was constant along the fault and decreased normal to the fault with a half width proportional to rupture propagation distance. He observed that reduction of elastic constants in the damaged region impeded propagation of the rupture, but he did not explicitly relate the damage to fracture energy.

[7] *Dalguer et al.* [2003a, 2003b] performed a three-dimensional (3-D) dynamic calculation of shear rupture on a preexisting fault in a medium in which tensile cracks could be generated. In each element of the calculational mesh, opening displacement could occur following a stress-strain path in which prescribed fracture energy was absorbed. Tension cracks formed on the extensional side of a propagating shear rupture. Abrupt stopping of rupture caused further tensile crack propagation in a flower-like structure radiating from the edge of the shear rupture.

[8] Variable fracture energy on the fault plane is implied by variation of D_c , the slip displacement at the end of the breakdown process. *Ohnaka* [2003] observed in laboratory friction experiments that D_c is proportional to λ_c , the predominant wavelength of irregularity on the surface. He fits laboratory data on both friction and fracture and inferred data for earthquakes by a single relation in which D_c is proportional to λ_c times a power of relative stress change. *Abercrombie and Rice* [2004] seek to explain increase of stress drop and fracture energy with earthquake size by a universal slip-weakening law in which stress drop continues to increase as a small power of slip up to the largest slips.

[9] *Poliakov et al.* [2002] examine the stress field near the tip of a steadily propagating semi-infinite crack with a slip-weakening friction law and show that stresses promoting off-fault failure or branching increase with propagation velocity. Near-fault failure may occur even if the far-field stress state is not conducive to continued propagation of a fault branch. They suggest that repeated initiation of failed branches is a mechanism to increase fracture energy. *Z. Reches and T. A. Dewers* (Gouge formation by dynamic pulverization during earthquakes, submitted to *Earth and Planetary Sciences Letters*, 2004) observe that fault gouge has an extremely fine particle size distribution, which they explain by comparing the *Poliakov et al.* [2002] predictions with shock wave data on pulverization. *Kame et al.* [2003] calculate dynamic fault branching. *Rice et al.* [2004] extend the results of *Poliakov et al.* [2002] to a steadily propagating slip pulse with finite slipping length. They show contours of a region near the crack tip in which a Coulomb failure criterion is met and a smaller region in which absolute tensile values of stress are reached.

[10] In this work I will examine neither a semi-infinite crack nor a slip pulse, but a crack with uniform stress drop growing from a point. The propagation distance of the crack will be a fundamental scaling parameter.

[11] *Andrews* [1976] calculated a mode 3 rupture in which there was no preferred fault plane. The material was uniform with a constitutive law that was linear elastic up to a yield stress and then yielded with strain weakening. Despite the

localizing tendency of strain weakening, the stress concentration at the rupture front produced yielding in a damage zone with thickness proportional to propagation distance. Fracture energy was also proportional to propagation distance. The solution was self-similar with constant propagation velocity somewhat less than the limiting velocity for a crack.

[12] In this paper I will calculate a mode 2 rupture with a preferred fault plane. Stress drop on the fault occurs as abruptly as can be well-resolved by the numerical discretization. Yielding off the fault will not be strain weakening but will be Coulomb yielding with constant internal friction. As in work by *Andrews* [1976], I will again find a self-similar solution with fracture energy proportional to rupture propagation distance.

2. Numerical Calculations

[13] A calculation is performed here of a spontaneously propagating rupture with uniform stress drop on a fault plane in a medium that yields plastically off the fault when the stress state reaches a Coulomb yield condition. Slip on the fault plane is calculated using the traction-at-split nodes (TSN) boundary condition [*Andrews*, 1999]. Yielding off the fault plane is calculated by the stress glut (SG) method [*Andrews*, 1999], which is simply an adjustment of stress to meet a yield condition. The SG method can be applied without recourse to equivalent body force if one does not use a finite element method, in which displacement and force are the fundamental dependent variables, but a method in which velocity and stress are the fundamental variables. It is also necessary to have all components of the stress tensor defined at common points. Such a method could be developed in 3-D using tetrahedral elements without a stiffness matrix. In this paper I use a 2-D method with triangular elements [*Andrews*, 1973]. This paper is an extension of earlier work in antiplane strain [*Andrews*, 1976] to plane strain.

[14] Working in 2-D allows very fine numerical discretization. The element size is 2 m, and the time step is 0.0003167 s. The dynamic solution for a bilateral rupture is examined to a distance of 2 km from the initiation point. The grid is extended to large enough distances that no artificial reflections contaminate the plotted results.

[15] In order to make informative comparisons, four calculations are performed. In the first calculation, a slip-weakening model with constant fracture energy is used on the fault plane, with the off-fault response being elastic. The second calculation has nonelastic response off the fault, while the stress drop on the fault occurs as abruptly as can be adequately resolved by the numerical discretization. The third calculation uses slip velocity from the second calculation as a boundary condition on a fault in an elastic medium. The calculated shear traction on the fault gives a slip-weakening model with variable fracture energy that is seismologically equivalent to the model with nonelastic off-fault response. The fourth calculation shows that artificially limiting slip velocity on a fault in an elastic medium can mimic the effect of off-fault yielding.

[16] In all calculations the medium is uniform with density $\rho = 2700 \text{ kg/m}^3$, P wave speed $v_P = 5196 \text{ m/s}$, and S wave speed $v_S = 3000 \text{ m/s}$. No damping or artificial viscosity is used; the only mechanism to control numerical oscillation generated at the rupture front is to have the

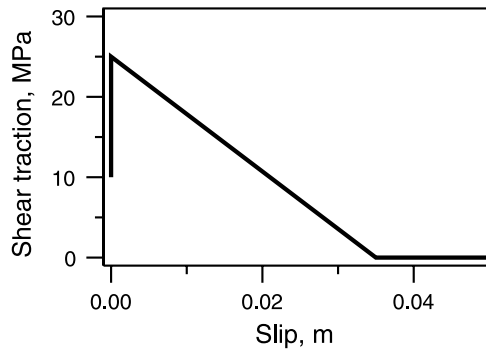


Figure 1. Slip-weakening friction law.

breakdown zone spread over a number of discretization intervals. The 2-D coordinate system is oriented with the x axis along the fault and the y axis normal to it.

[17] In order to maximize the thickness of the zone subject to plastic yielding, I have arbitrarily chosen a relatively small effective pressure, appropriate to about 3 km depth, and a large stress drop of 10 MPa. The initial stress state is uniform with $\sigma_{xx} = \sigma_{yy} = -50$ MPa (stress is negative in compression) and $\sigma_{xy} = 10$ MPa. The coefficient of static friction on the fault is 0.5, and the coefficient of kinetic friction is zero. In the calculations with elastic response off the fault, symmetry assures that normal stress on the fault does not change, so that slip starts spontaneously when shear traction rises from 10 to 25 MPa, and shear traction falls to 0 MPa at the conclusion of a breakdown process. Static friction is large enough that rupture does not jump to supershear velocities. Rupture does not stop in these models with uniform initial stress.

2.1. Slip-Weakening Calculation

[18] Use of a slip-weakening friction law, in which friction is a continuous function of slip, eliminates the singularity present in a sharp-tipped crack solution and makes numerical rupture calculation possible. Let D_c be the interval of slip over which friction drops from its static to its kinetic value. A numerical solution can be accurate if the cohesive zone width l , the distance in which slip

changes from zero to D_c , is sufficiently larger than the discretization interval. Unfortunately, l is not constant in a dynamic solution. In the case of a bilateral crack-like rupture with uniform stress drop $\Delta\tau$, cohesive zone width l at a distance L from the initiation point is approximately

$$l = (\mu/\Delta\tau)^2 D_c^2 / L \quad (1)$$

where μ is shear modulus [Andrews, 2004]. For constant D_c and $\Delta\tau$, cohesive zone width l varies inversely with propagation distance L .

[19] (In analytic treatments of steady propagation of a semi-infinite crack or a slip pulse, there is a Lorentz contraction that shrinks the length scale toward zero as the limiting rupture speed is approached. In this case of unsteady growth of a crack, rupture speed increases as length increases, such that the shrinking cohesive zone given above is consistent with Lorentz contraction.)

[20] The slip-weakening friction law used in this calculation is shown in Figure 1. The value of $D_c = 0.035$ m is chosen such that l predicted by equation (1) is 7.2 m (3.6 discretization intervals) at a propagation distance of $L = 1000$ m with a stress drop of 10 MPa. A value of D_c this large (larger than laboratory values) is required for numerical accuracy at $L = 1000$ m, even with a grid interval as small as 2 m. The cohesive zone is less well resolved at larger propagation distances, while at smaller propagation distances it is better resolved than it needs to be, and l becomes on the order of L at the nucleation patch size, which is roughly 100 m in this case.

[21] Rupture could be initiated by imposing a stress drop in a patch with half length 100 m or larger, but in this calculation I took some care to find a close approximation, shown in Figure 2, to a critical state that is on the verge of dynamic instability. The slip function shown, which was found by trial, produces static stress shown by the black curve. This was chosen as the initial state for the dynamic calculation. The red curve shows stress given by the slip-weakening law, to which stress drops at the beginning of the dynamic calculation. Rupture grows and accelerates smoothly from this state.

[22] Figure 3 (left) shows calculated results as a function of time at a point on the fault 1000 m from the initiation

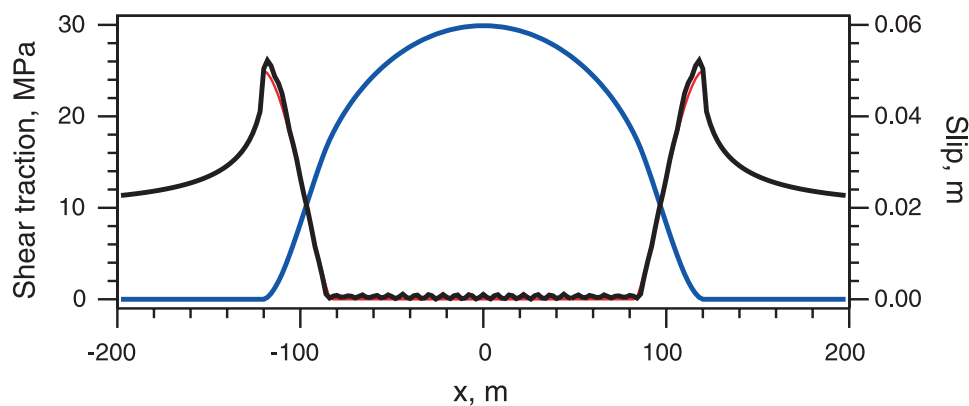


Figure 2. Quasi-static solution (found by trial) used as initial state in the dynamic slip-weakening calculation. Blue curve indicates slip; black curve indicates shear traction σ_{xy} in static solution; red curve indicates shear traction σ_{xy} from slip-weakening law.

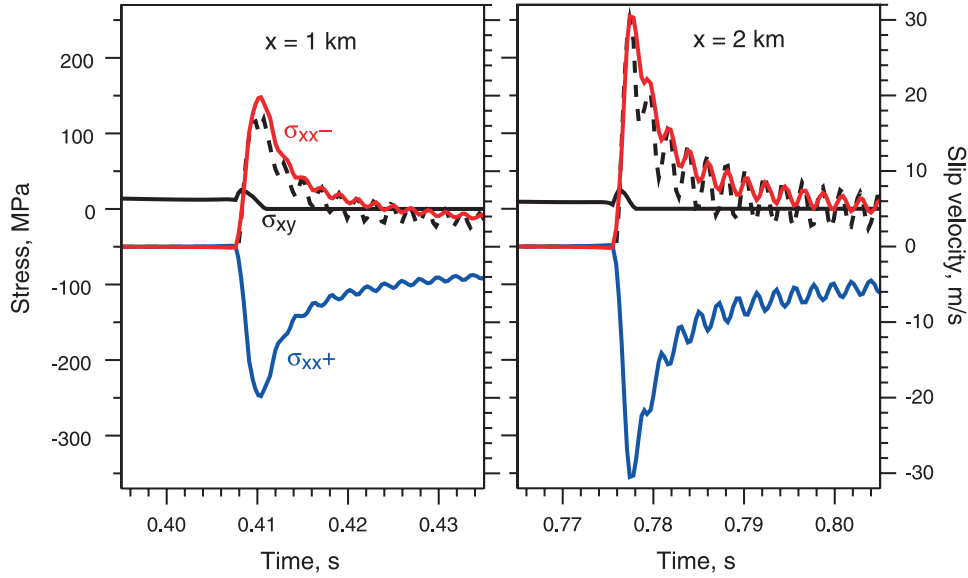


Figure 3. Time histories from the slip-weakening calculation. (left) At distance 1000 m from the initiation point. (right) At 2000 m from the initiation point. Black solid curve indicates shear traction σ_{xy} ; dotted curve indicates slip velocity (scale on right); blue curve indicates σ_{xx} in adjacent element on compressive side of fault; red curve indicates σ_{xx} in adjacent element on extensional side of fault.

point. Shear traction on the fault, σ_{xy} , after rising to a peak value of 25 MPa, falls to zero in a time interval of 0.0027 s, which corresponds to a cohesive zone length of 7.2 m, as predicted by equation (1). Normal traction on the fault, σ_{yy} , does not change from its initial value of -50 MPa. The variables σ_{xy} , σ_{yy} , and slip velocity are defined at the split node in the TSN method. The other component of stress, σ_{xx} , does not enter into the frictional calculation. It changes in opposite senses on opposite sides of the fault. Values of σ_{xx} defined in interiors of adjacent elements on either side of the fault (within 1 m of the fault surface) are plotted in Figure 3. These components change by 200 MPa, much larger than the 10 MPa drop of shear traction. An absolute tension of 150 MPa is reached on the extensional side of the fault. Slip velocity is approximately proportional to the change of σ_{xx} , because each is proportional to strain in the case of steady propagation.

[23] As the cohesive zone l shrinks, average strain in the cohesive zone $\epsilon_{xx} = D_c/l$ increases, so peak values of slip velocity and σ_{xx} will increase. This can be seen in Figure 3 (right), which shows time histories at a distance of 2000 m from the initiation point. Shear traction drops in a shorter time interval, but the interval is a bit larger than half that at 1000 m, because of discretization error. The error is evidenced by increased high-frequency oscillation in other variables following the rupture front. Peak values of change of σ_{xx} and of slip velocity are a bit smaller than the theoretical prediction of twice the values at 1000 m.

[24] On a fault with a slip-weakening friction law in an elastic medium, a rupture with constant drop of shear traction will produce values of σ_{xx} that increase without limit as the rupture grows. Such a model is not physical, because the stress response of any real material will become nonelastic at sufficiently large shear or tensile stress. The

expected response to σ_{xx} is microcracking at orientations different from the main fault plane.

2.2. Nonelastic Off-Fault Response

[25] In the nonelastic calculation the material in which the fault is embedded has a Coulomb yield criterion [Scholz, 1990]. The calculated thickness of the zone that yields is much larger than the grain size of rocks, so a macroscopic failure criterion is appropriate. In this 2-D plane strain calculation, the maximum and minimum principal stress directions are assumed to be in the x - y plane. The intermediate principal stress σ_{zz} does not enter the calculation. The resolved shear stress maximized over all orientations is

$$\tau = \sqrt{\sigma_{xy}^2 + [(\sigma_{xx} - \sigma_{yy})/2]^2} \quad (2)$$

For an internal coefficient of friction of $\tan \phi$, the Coulomb criterion is

$$\tau \leq c \cos \phi - (1/2)(\sigma_{xx} + \sigma_{yy}) \sin \phi \quad (3)$$

where c is cohesion. The minus sign on the right-hand side arises because the sign convention for stress is positive in tension.

[26] In each time step of the numerical calculation, stress components are first incremented elastically. Then, if the Coulomb criterion is violated, stress components are adjusted as explained in Appendix A, so that equality holds in the criterion. The adjustment is constrained so that there is no change in mean compressive stress $(\sigma_{xx} + \sigma_{yy})/2$. This means that inelastic volumetric strain (dilatation or compaction) is zero. The shear stress components σ_{xy} and $(\sigma_{xx} - \sigma_{yy})/2$ are reduced by a common factor (so that the increment of plastic strain is in the direction of stress deviators in

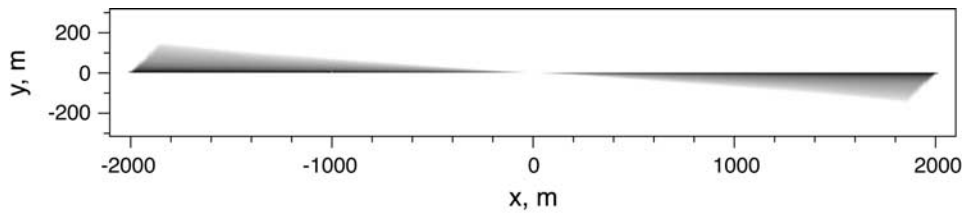


Figure 4. Plastic strain at time 0.785 s. Darkest shading indicates magnitude of plastic strain equal to 0.0016.

stress space) to meet the Coulomb criterion. The adjustment to each stress component $\delta\sigma_{ij}$ is converted to an increment of the plastic strain tensor $\Delta\epsilon_{ij}^p = \delta\sigma_{ij}/\mu$. Note that division by μ rather than 2μ means that plastic strain is engineering shear strain rather than tensor shear strain. Plastic strain is volume density of seismic potency. The increment of work done irreversibly per unit volume is

$$\Delta w = \frac{1}{2} \sum_{ij} \sigma_{ij} \Delta\epsilon_{ij}^p. \quad (4)$$

[27] Using the implementation of plasticity as described above, I found that the distribution of plastic strain was mildly unstable as the grid was refined. The calculation is stabilized by introducing time-dependent relaxation of the stress state toward the yield condition (Maxwellian viscoplasticity). See Appendix A for the numerical implementation. I found that using a relaxation time as short as the time for an S wave to propagate one grid spacing was enough to achieve stability. A perturbation of cohesion in a small spatial region had no effect on the plastic strain distribution at distant points, so I conclude that there is no physical instability.

[28] The internal coefficient of friction chosen for this calculation, $\tan \phi = 0.75$, is larger than the static coefficient of friction on the fault, 0.5, so that the fault is a preferred plane of weakness. I choose cohesion to be zero, $c = 0$. The calculation of frictional fault slip in section 2.1 depends on the chosen stress drop of 10 MPa but is otherwise independent of the initial stress state. The calculation with Coulomb yielding, however, depends on all components of the initial stress state. I have arbitrarily chosen $\sigma_{xx} = \sigma_{yy}$ with $\sigma_{xy} \neq 0$, which implies that the initial maximum compressive principal stress direction is at 45° to the fault. Clearly, this parameter needs to be varied in subsequent work.

[29] The calculation of frictional slip on the fault itself, in this case, does not use a slip-weakening law. Instead, I imagine that the transition from static to kinetic friction occurs quickly, and the rate of the breakdown process is limited only by the requirement of numerical resolution. After shear traction on the fault reaches the static friction level, friction decreases to the kinetic value in a time interval of 0.0035 s. In this time interval an S wave propagates 10.5 m, or 5.25 grid intervals, which is sufficient to keep artificial numerical oscillation relatively small. Results calculated with such a time-weakening law may be seen in Andrews [2004], in which fracture energy on the fault increases with the square root of rupture propagation distance. That result will be seen to be different when there

is interaction with off-fault yielding. Other input parameters are the same as in the previous calculation.

[30] In this and the following calculations, rupture is artificially initiated from the uniform state by prescribing that stress drops behind a rupture front that spreads from the origin at a speed of 2000 m/s. After the rupture achieves a sufficient length, it propagates spontaneously at greater velocity. A justification of this initiation procedure will be given below.

[31] Figure 4 is a snapshot at time 0.785 s of the magnitude of plastic strain,

$$\epsilon^p = \sqrt{[(\epsilon_{xx}^p - \epsilon_{yy}^p)/2]^2 + (\epsilon_{xy}^p)^2}. \quad (5)$$

Material has yielded only on the extensional side of the fault. The magnitude of plastic strain has a uniform value of $\epsilon^p = 0.0016$ adjacent to the fault, and it decreases away from the fault with a thickness proportional to rupture propagation distance. The cross section of the plastic strain distribution is shown in Figure 5 with fault-perpendicular distance y normalized by propagation distance x . The zone of plastic strain has a width at half maximum of about 0.02 times propagation length. Within that width, along-strike extension predominates, but the two components of plastic strain are comparable at larger widths. Both drop to zero at a width 0.08 times propagation length.

[32] Figure 6 (left) shows time histories at 1000 m from the initiation point. Shear traction on the fault, σ_{xy} , falls from its peak value to zero in a time interval of 0.0035 s, as prescribed by the time-weakening law. This time interval is

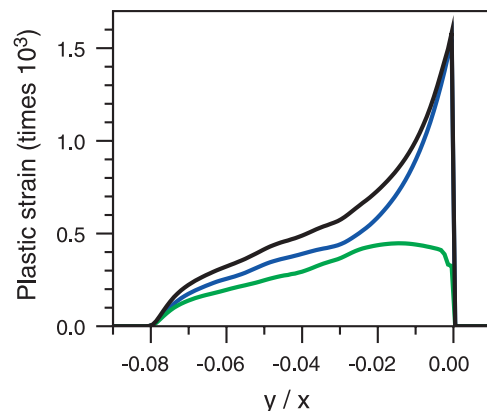


Figure 5. Cross section of plastic strain distribution. Black curve indicates magnitude; blue curve indicates component $(\epsilon_{xx}^p - \epsilon_{yy}^p)/2$; green curve indicates component ϵ_{xy}^p .

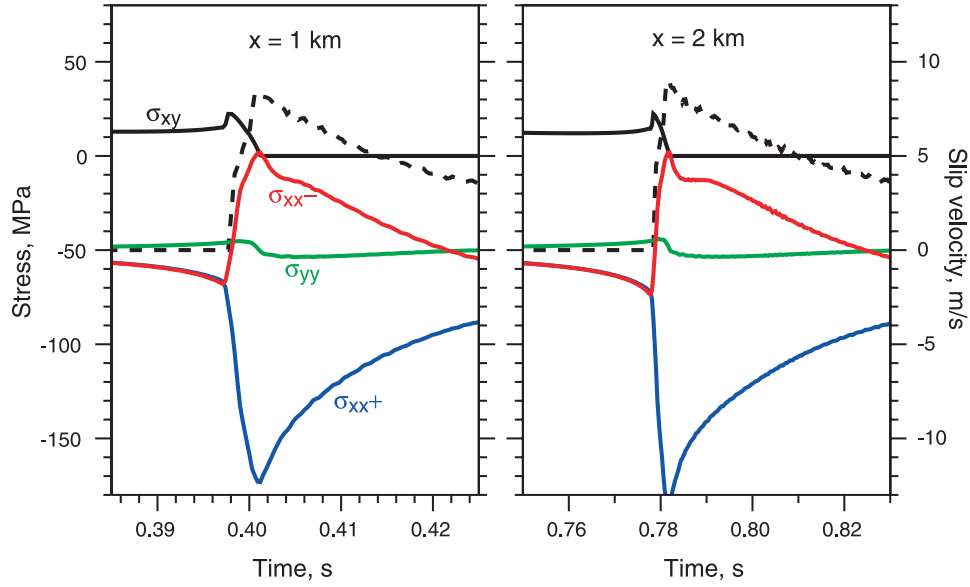


Figure 6. Time histories in the calculation with off-fault yielding. (left) At distance 1000 m from the initiation point. (right) At 2000 m from the initiation point with the timescale expanded by a factor of 2. Black solid curve indicates shear traction σ_{xy} ; dotted curve indicates slip velocity (scale on right); blue curve indicates σ_{xx} in adjacent element on compressive side of fault; red curve indicates σ_{xx} in adjacent element on extensional side of fault. green curve indicates normal traction σ_{yy} on fault.

not indicative of the cohesive zone width, because plastic yielding is occurring through a significant fraction of the time window plotted. Because yielding on one side of the fault breaks the symmetry, normal traction on the fault, σ_{yy} , changes from its initial value. It is more tensile at the onset of slip, and it becomes more compressive while yielding is occurring. (The change of normal stress is accounted for in the calculation of slip on the fault.) Change of normal stress is smaller than changes of other stress components, because the fault still warps in such a way as to reduce its change. The stress component σ_{xx} on the extensional side of the fault changes in a tensile sense, but yielding in shear prevents it from reaching absolute tensile values. The peak value of slip velocity is 8 m/s.

[33] Time histories of the same variables at 2000 m from the initiation point are shown in Figure 6 (right). The timescale is changed by a factor of 2, so that the time interval shown is twice as long. Shear traction, σ_{xy} , falls in the same time interval as before. It appears to fall more abruptly because of the change of time scale. The curves for the other variables are very similar in Figure 6. The peak value of slip velocity is still 8 m/s. The decay of slip velocity and σ_{xx} after their peaks is slower by a factor of 2. Aside from the prescribed behavior of σ_{xy} on the fault, the solution is close to being self-similar, with the timescale expanding in proportion to propagation distance.

[34] Energy absorbed near the rupture front can be separated into two parts. Energy absorbed per unit area on the fault plane, minus the frictional work done against the sliding frictional stress τ_k , is

$$W_{\text{on}} = \int (\sigma_{xy} - \tau_k) \Delta v dt \quad (6)$$

where Δv is slip velocity. In this case $\tau_k = 0$. Energy absorbed off the fault plane, projected onto the fault, per unit area of fault, is

$$W_{\text{off}} = \int w dy \quad (7)$$

where w is summed from increments defined in equation (4).

[35] Figure 7 shows energy absorbed on and off the fault and the sum as a function of distance along strike. The contribution on the fault W_{on} approaches a constant value, because slip velocity in the cohesive zone approaches a constant value in this case, and the time duration of the

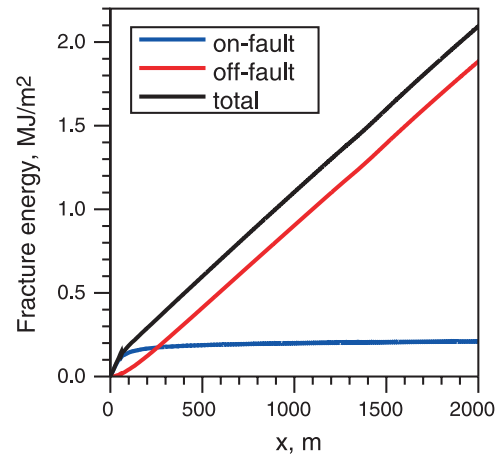


Figure 7. Energy absorbed near the rupture front in the nonelastic calculation. Blue curve indicates energy absorbed on the fault; red curve indicates energy absorbed off the fault; black curve indicates total.

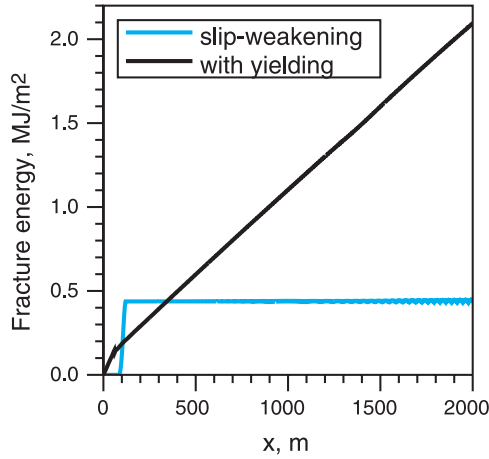


Figure 8. Comparison of fracture energy as a function of rupture distance. Black curve indicates total energy absorbed near the rupture front in the nonelastic calculation; cyan curve indicates fracture energy in the slip-weakening calculation.

cohesive zone is prescribed to be constant. If the numerical discretization were refined, and the time-weakening duration decreased, this contribution would be smaller. The contribution from off the fault W_{off} , after the forced initiation, increases linearly with propagation distance.

[36] The linear increase of energy absorbed per unit area is consistent with a self-similar solution. Results shown in Figure 6 suggest a scaling in which stress and velocity remain invariant and the timescale is proportional to the length scale. Under such a scaling, energy per unit volume, which has the same dimensions as stress, would be invariant, so energy per unit area would be proportional to length. I conclude that in the limit of vanishing numerical grid size the solution for a rupture with constant abrupt stress drop with Coulomb yielding off the fault is self-similar. If one wanted to follow the solution over a broad range of length scales, one could use the remapping scheme of *Aochi and Ide* [2004]. The forced initiation I used is intended to approximate self-similar growth at small scales.

[37] In order to examine the dependence of rupture velocity on energy loss, it is informative to compare the nonelastic and the slip-weakening calculations. Figure 8 compares fracture energy from the slip-weakening rupture (which is propagating spontaneously beyond the nucleation distance of 100 m) with the total energy loss in the nonelastic rupture (which is propagating spontaneously beyond 60 m). Figure 9 compares rupture speeds in the two calculations. These are found by differencing tabulated arrival times, smoothing over 20 time steps, and plotting velocity as a fraction of Rayleigh speed. Since the cohesive zone shrinks as the rupture grows in the slip-weakening case, the trailing edge of the cohesive zone propagates faster than the leading edge. Both approach the Rayleigh speed as the rupture lengthens. Rupture velocity in the nonelastic case is slower at distances where the total energy loss is larger. Because the plastic strain distribution is self-similar, the trailing edge of the cohesive zone propagates at a constant fraction of the velocity of the leading edge.

[38] Energy loss due to yielding off the fault affects the rupture velocity. Let us examine whether it is appropriate to define fracture energy as

$$G = W_{\text{on}} + W_{\text{off}} \quad (8)$$

in regard to the determination of rupture velocity, despite the fact that this G is not related to a constitutive law on the slip plane.

[39] Theoretical predictions of rupture velocity from linear fracture mechanics are conventionally expressed in terms of the ratio of fracture energy to the energy released by equivalent expansion of a static crack. Consider the slip function of a static plane strain shear crack,

$$\Delta u = \frac{3}{2} \frac{\Delta \tau}{\mu} (L^2 - x^2)^{1/2} \quad (9)$$

(For a dynamic crack with instantaneous length L the coefficient is smaller than its static value $3/2$, and it is 1.09 for a crack propagating near the Rayleigh speed.) The static energy of the crack, the virtual work done in creating the crack with stress τ_k in a uniform stress field τ_0 , is

$$U = -(1/2)(\tau_0 + \tau_k) \int \Delta u dx \quad (10)$$

and the work done against friction is

$$W = \tau_k \int \Delta u dx \quad (11)$$

The energy available to be absorbed in the breakdown process and to be radiated away is

$$E = -U - W = (1/2)(\tau_0 - \tau_k) \int \Delta u dx = \frac{3\pi}{8} \frac{(\Delta \tau)^2}{\mu} L^2 \quad (12)$$

As the crack lengthens by an increment dL , the energy absorbed in the breakdown process is $2GdL$ (accounting for

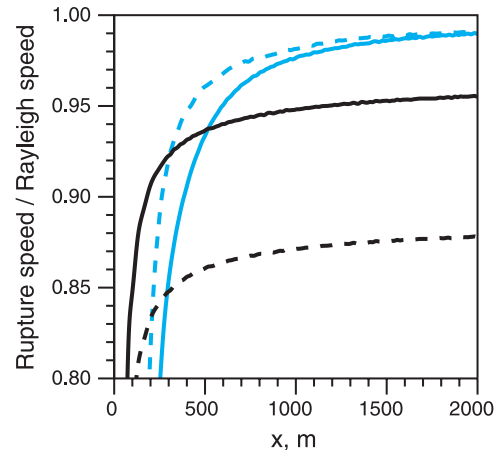


Figure 9. Rupture velocity as a fraction of Rayleigh speed. Black curves indicates nonelastic calculation; cyan curves indicates slip-weakening calculation. Solid curves indicates leading edge of cohesive zone; dashed curves indicates trailing edge of cohesive zone.

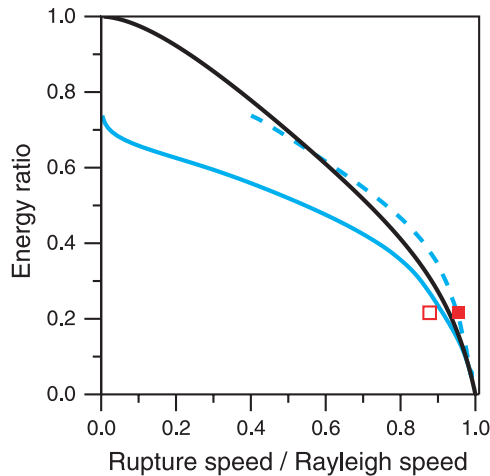


Figure 10. Ratio of fracture energy to available energy released plotted against rupture velocity normalized by Rayleigh speed. Black curve indicates theoretical; solid cyan curve indicates slip weakening, leading edge; dashed cyan curve indicates slip weakening, trailing edge; solid red square indicates leading edge, nonelastic calculation at 2 km; open red square indicates trailing edge, nonelastic calculation at 2 km.

both ends of the bilateral crack), and the increment of available energy is

$$dE = \frac{3\pi}{4} \frac{(\Delta\tau)^2}{\mu} L dL \quad (13)$$

which is proportional to L .

[40] Energy absorbed at the tip of a self-similarly growing plane strain crack may be found by evaluating equations (1.2), (3.6), and (3.8) of *Kostrov* [1964]. Energy absorbed is proportional to crack length, and its ratio to energy available (13) is a function of rupture velocity,

$$2G/(dE/dL) = g(v) \quad (14)$$

The theoretical function $g(v)$ is plotted as the solid black curve in Figure 10. *Freund* [1989] shows that $g(v)$, which he terms the dynamic energy release rate, but might more appropriately be called the dynamic energy absorption rate, is a universal function of rupture velocity, applicable to unsteady crack propagation. *Nielsen and Madariaga* [2003] find a smaller dynamic energy absorption rate for a self-healing slip pulse; the solution here is appropriate if friction does not recover from the sliding value.

[41] Fracture energy in the slip-weakening calculation is constant, but its ratio to energy released is a decreasing function of crack length. This ratio is plotted in Figure 10 as a function of the velocities of the leading and trailing edges of the cohesive zone, as the solid and dashed cyan curves respectively. As the cohesive zone shrinks, these velocities of a slip-weakening crack approach the theoretical prediction for a sharp-tipped crack.

[42] If fracture energy is proportional to crack length, rupture velocity is predicted to be constant. Also plotted in Figure 10 are points from the nonelastic calculation at $L =$

2000 m of $2G/(dE/dL) = 0.216$ versus calculated velocities of the leading and trailing edges of the yielding zone. These points fall close to the theoretical prediction and are close to the paths followed by the slip-weakening calculation. From this agreement we may conclude that energy absorbed in the breakdown process off the fault contributes to the fracture energy that determines rupture velocity.

2.3. Equivalent Fault Friction in an Elastic Medium

[43] Slip velocity in the nonelastic calculation has a smaller peak value than it would have without the off-fault yielding, and it has a slower rate of decay after the peak, despite the rapid drop in shear traction on the fault. This section examines the question of what friction law on a fault in an elastic medium would produce this solution for slip velocity.

[44] Slip velocity as a function of space and time is saved from the nonelastic calculation, and it is used in a new calculation as a boundary condition on a fault in an elastic medium. The output of this calculation, in addition to the solution in the elastic medium, is shear traction on the fault that is consistent with the given slip velocity function.

[45] Figure 11 shows shear traction σ_{xy} as a function of slip at two points, 1 km and 2 km from the initiation point. The curves are very close to being self-similar, with the slip scale stretched in proportion to propagation distance. The slip-weakening curves are concave upward, which is consistent with peak slip velocity occurring near the beginning of the breakdown zone. A slip-weakening curve that is straight or convex upward produces peak slip velocity at the end of the breakdown zone [*Ida*, 1972].

[46] Peak shear traction is slightly larger than its previously prescribed peak of 25 MPa, and shear traction is not zero beyond the end of the breakdown zone. It is remarkable, however, that peak and residual stress values closely approximate those in the previous problem. Because of the small long-period oscillation of baseline stress, there is some ambiguity in using equation (6) to get fracture energy.

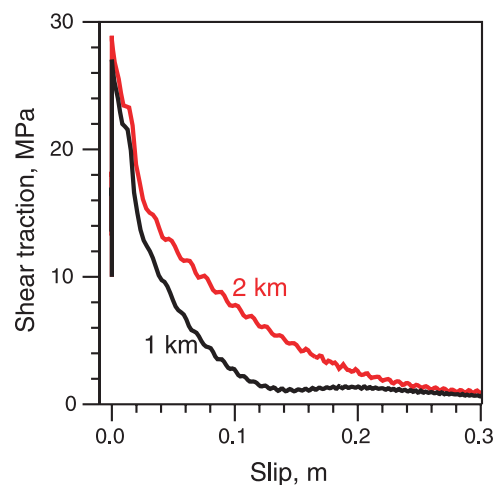


Figure 11. Shear traction as a function of slip from the calculation using slip velocity from the nonelastic calculation as a boundary condition on a fault in an elastic medium. Black curve: 1000 m from the initiation point; red curve: 2000 m from the initiation point.

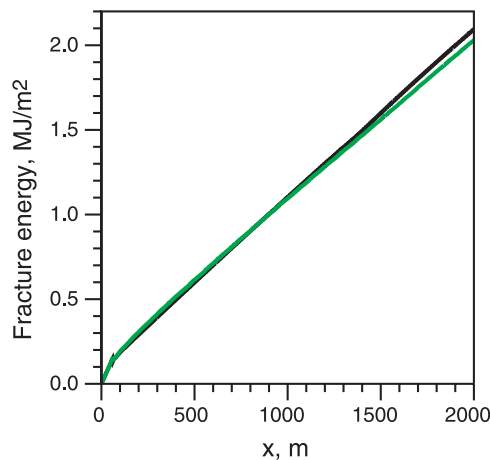


Figure 12. Fracture energy as a function of distance. Black curve indicates nonelastic calculation; green curve indicates calculation with slip-velocity boundary condition.

The minimum of shear traction at 1 km occurs at about the time yielding ends at that distance, and slip velocity at that time is 0.4 of its peak value. I calculated fracture energy at all distances by stopping the integration in equation (6) when slip velocity dropped below 0.4 of its peak value. The result for fracture energy as a function of distance shown in Figure 12 agrees remarkably well with the nonelastic calculation, considering the ambiguity of its determination. Therefore this slip-velocity function predicts approximately constant peak and sliding stress, predicts a well-localized cohesive zone that grows self-similarly, and it contains information on fracture energy, which determines rupture velocity.

[47] A nominal value of equivalent D_c is found by dividing fracture energy by 12.5 MPa (half the difference between peak stress and sliding stress). Figure 13 is a snapshot at time 0.785 s of the calculation with the slip-velocity boundary condition, comparing D_c with total slip at that instant. D_c at the rupture front is 16 percent of slip at the rupture origin.

[48] One could reproduce this solution using a slip-weakening law with breakdown slip D_c dependent on position, as was done by *Aochi and Ide* [2004]. In that way one could simulate the effect of off-fault yielding using an elastic method. Unfortunately, D_c would not be a constitutive property of each point on the fault; it would depend on the location of the initiation point, on the assumption of uniform stress drop, and on the magnitude of the stress drop. A more generally applicable method is given in section 2.4.

2.4. A Velocity-Toughening Model

[49] In an approximately steadily propagating rupture front, slip velocity is proportional to the strain component that is the derivative of slip with respect to propagation direction. Stress components other than shear traction on the slip plane depend on this strain component in an elastic medium, and they can become large if the cohesive zone shrinks. If stress change is limited by nonelastic response, then slip velocity is limited. *Andrews* [2004] demonstrated a converse effect, that imposing a limit on slip velocity limits stress change symmetrically across the fault, mimicking the

effect of symmetric off-fault yielding. Shear traction, altered to be consistent with the alteration of slip velocity, is such that the effective slip-weakening curve has increased D_c . Imposing a limit on slip velocity increases fracture energy, so it may be termed *velocity toughening*.

[50] The calculation in this section examines to what extent imposing a limit on slip velocity can match the effect of nonsymmetric Coulomb yielding in a plane strain shear rupture. A calculation is done using the same time-weakening prescription on the fault as in the nonelastic calculation with the addition of an imposed limit on slip velocity. Material response off the fault is elastic. The limit on slip velocity is arbitrarily chosen to be 8 m/s, which was the peak value in the nonelastic calculation. When slip velocity is at its limiting value, a consistent value of shear traction σ_{xy} is found by inverting equation (4) of *Andrews* [1999] to find traction consistent with prescribed velocity rather than velocity consistent with prescribed traction. Calculated results are a bit noisy, so perhaps the numerical procedure is not optimum.

[51] Figure 14 shows time histories at 1000 m and at 2000 m from the initiation point. Scales are the same as in Figure 6 for the nonelastic calculation, and the timescale is changed by a factor of 2 at the larger distance. The solution is approximately self-similar. Shear traction decays more slowly than the time-weakening prescription. The slow decay of σ_{xy} increases fracture energy, since it increases overlap of the pulses of shear traction and slip velocity. The stress component σ_{xx} changes symmetrically in opposite senses across the fault, and the changes are proportional to slip velocity. The size of the change is intermediate between the nonsymmetric changes in the nonelastic calculation.

[52] Figure 15 shows shear traction as a function of slip at distances 1000 m and 2000 m in the velocity-toughening calculation. Notice that D_c is proportional to distance. The shapes of these curves resemble, although they are not the same as, the shapes of the slip-weakening curves in Figure 11.

[53] Fracture energy in the velocity-toughening calculation increases linearly with rupture propagation distance after the forced initiation (Figure 16). It is smaller than fracture energy from the nonelastic calculation, in the ratio 1/1.4. *Andrews* [2004] shows that D_c and fracture energy in

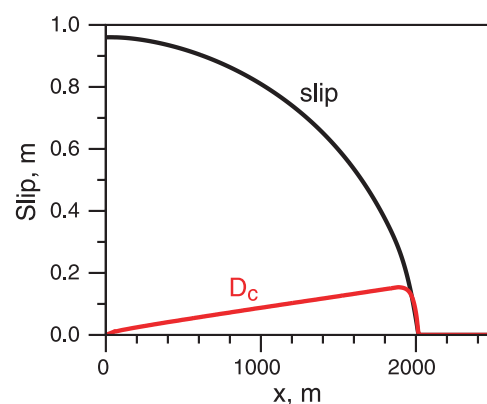


Figure 13. Solution calculated with slip-velocity boundary condition at time 0.785 s. Black curve indicates slip; red curve indicates equivalent D_c .

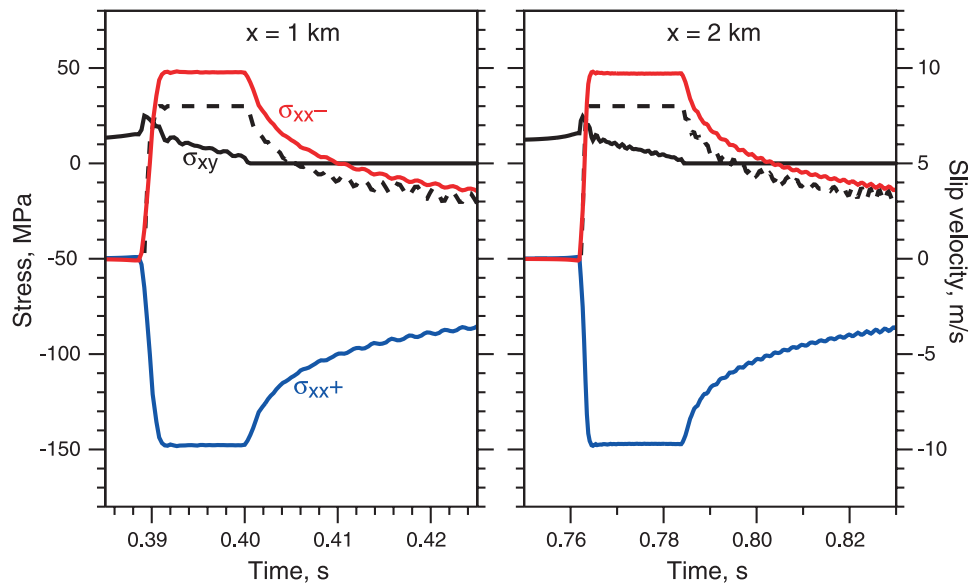


Figure 14. Time histories in the velocity-toughening calculation. (left) At distance 1000 m from the initiation point. (right) At 2000 m from the initiation point with the timescale expanded by a factor of 2. Black solid curve indicates shear traction σ_{xy} ; dotted curve indicates slip velocity (scale on right); blue curve indicates σ_{xx} in adjacent element on compressive side of fault; red curve indicates σ_{xx} in adjacent element on extensional side of fault.

a velocity-toughening model are inversely proportional to the limiting slip velocity. Therefore fracture energy in the velocity-toughening calculation could be adjusted to match the nonelastic calculation by changing the limit on slip velocity to 5.7 m/s. If one's only objective is to match fracture energy and have a cohesive zone of roughly the same dimension, then using the velocity-toughening model in an elastic medium is a satisfactory substitute for doing a nonelastic calculation.

[54] This calculation may be regarded as a calibration that shows that a limiting slip velocity of 5.7 m/s is generally applicable for a plane strain shear (mode 2) rupture at a confining pressure of 50 MPa and with off-fault Coulomb friction of 0.75. Since the velocity limit is directly related to the stress change limit, it may be considered a property of a location on a fault. Results calculated with a velocity-toughening model may be reasonably expected to be physically realistic for different ruptures with heterogeneous stress drop and with arbitrary initiation points. Fracture energy at a given point will be different in these different events, but the velocity limit, which is related to the stress state, is a point property.

3. Discussion

3.1. Dilatancy

[55] In the absence of off-fault yielding, the rupture considered in this paper produces tensile strain and absolute tensile values of stress on the extensional side of the fault. Such large extensional strain in a real material would produce nonelastic extensional strain (dilatancy) in the form of tensile microcracks, if not massive tensile failure.

[56] The plastic flow law I used with the Coulomb yield condition in this paper does not allow plastic volumetric strain (dilatancy or compaction). Only deviatoric or shear components of plastic strain are allowed. Yielding in shear

had the effect in the calculation that stress never reached absolute tensile values. As effective pressure approached zero, plastic shear strain was effective in limiting stress change. The plastic flow law used here needs to be supplemented with a law allowing plastic extensional strain when stress is tensile, but that condition was never reached in this calculation.

[57] A more realistic plastic flow law would allow some dilatancy to occur when all principal stress components are compressive, but its prescription is not trivial. Dilatant strain by itself in a compressive stress field will do work rather than absorb energy, so it must be intimately associated with the energy loss of plastic shear strain. A densely packed

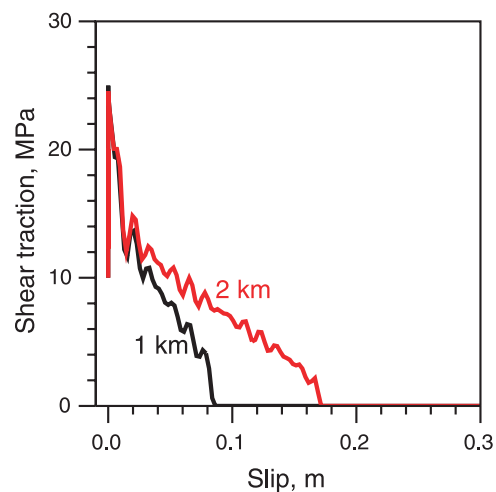


Figure 15. Shear traction as a function of slip from the velocity-toughening calculation. Black curve indicates 1000 m from the initiation point; red curve indicates 2000 m from the initiation point.

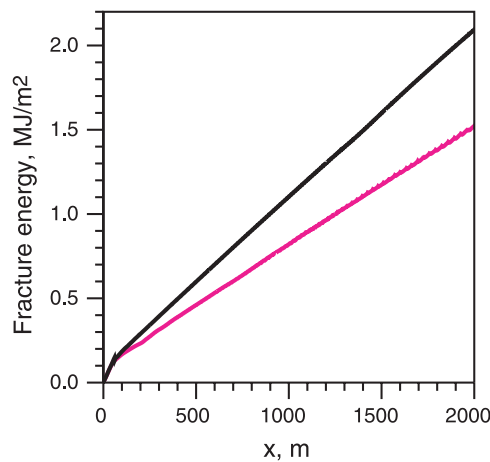


Figure 16. Fracture energy as a function of distance. Black curve indicates nonelastic calculation; magenta curve indicates velocity-toughening calculation.

granular medium will dilate when it is sheared, but as the plastic shear strain continues to increase the dilatant strain will approach a steady value that is dependent on confining stress [Morrow and Byerlee, 1989]. The damage zone of an established fault has been deformed nonelastically many times. It might be considered to be in a steady state of dilatancy, except for the fact that minerals can be precipitated in the pore space in the time interval between earthquakes. The expected amount of dilatancy in an earthquake is equal to the pore volume that was remineralized since the previous earthquake of the same size.

[58] Dilatancy is crucial in predicting the thermal pressurization resulting from shear heating in an earthquake. Dilatant strain reduces fluid pressure, and increased connectivity between pores can dramatically increase permeability, so that heated fluid can diffuse away, further reducing fluid pressure.

[59] This calculation also ignores the reduction of elastic constants that accompanies dilatant strain. Material that has suffered plastic shear strain in this model still has the same P and S velocities.

[60] Brune [2001] observes that granitic material in the damage zone of the San Andreas fault at Tejon Pass is powdered at an extremely fine scale, but no fault-parallel shear bands are evident. He interprets this texture as being produced by tensile failure. I suggest that it could be produced by plastic shear strain at other orientations with magnitude on the order of 0.001 in each earthquake.

3.2. Scaling With Depth

[61] A systematic study needs to be done, varying the parameters of the nonelastic calculation reported in this paper. I expect that the thickness of the zone of plastic yielding will be proportional to the square of stress drop and inversely proportional to the square of confining pressure. In order to maximize damage zone thickness in this calculation, I chose a large stress drop of 10 MPa and a relatively small confining pressure of 50 MPa. Stress drop was complete in this calculation, which one could imagine arises from thermal pressurization in a relatively impermeable medium.

[62] An effective pressure of 50 MPa with hydrostatic fluid pressure is appropriate to 3 km depth. Effective pressure increases linearly with depth. Observed stress drops are quite variable, but 10 MPa is a large value for any depth. Therefore I expect that the damage zone thickness and effective cohesive zone length will be smaller as depth increases.

[63] Nonelastic yielding has the effect of limiting the change of stress and of particle velocity near a rupture front. Conversely, limiting slip velocity at a rupture front in an elastic medium has the effect of producing fracture energy similar to the case with yielding. The limit on slip velocity that mimics off-fault yielding in a plane strain shear (mode 2) rupture is 5.7 m/s at 3 km depth. The slip-velocity limit is proportional to effective pressure. Therefore the limit is proportional to depth and will be 19 m/s at 10 km depth. Calibration calculations similar to these need to be done for mode 3 rupture. Andrews [2004] suggests that the velocity limit is roughly the same (certainly with a factor of 2) for mode 2 and mode 3 ruptures, so velocity toughening can be used in 3-D simulations.

[64] Limiting values of slip velocity that I propose here are rarely seen in numerical simulations; fine discretization is needed in order to reach such large values. My prediction of fracture energy resulting from off-fault yielding, and D_c from the equivalent velocity-toughening model, is larger than laboratory values, but not large enough to give resolvable breakdown zones with coarse numerical discretization.

3.3. Length Scaling

[65] Crack-like solutions growing from a point are considered in this work, and rupture propagation distance L appears as a fundamental parameter in scaling the results. How might this parameter be interpreted in more general problems? In a crack solution the square of the stress intensity factor is proportional to length, $K^2 \propto L$. A conjecture, that needs to be verified by further work, is that, in more general problems with heterogeneous stress drop, L may be replaced by K^2 in the scaling relationships. Then $G \propto K^2$. The dynamic energy absorption rate is also proportional to K^2 , so energy absorbed is a constant fraction of energy released at a given confining pressure. It remains to be seen how this ratio depends on confining pressure.

[66] When rupture length L exceeds width W , as when width is limited by seismogenic depth, stress intensity depends on W rather than on L . Therefore L in the predictions in this paper is limited by W .

[67] In an event with heterogeneous stress drop, I expect that the parameter L will be related to the length scale of the largest asperities. In a slip-pulse model, which in my opinion is just an idealized representation of the effect of heterogeneous stress drop, perhaps L will be the slip-pulse length.

4. Summary

[68] A fault zone consists of a thin principal slip zone surrounded by a damage zone. The stress concentration at a rupture front can produce nonelastic response with energy loss in the damage zone.

[69] In this work the nonelastic response is plastic shear strain when the stress state is at a Coulomb yield condition.

The internal coefficient of friction of the medium is $\tan \phi = 0.75$. The slip zone is modeled as a fault plane with a smaller static coefficient of friction, so that it is a preferred plane of weakness. The initial maximum compressive principal stress direction is at 45° to the fault.

[70] A crack-like dynamic solution is calculated in 2-D in which a mode 2 rupture grows from a point with uniform drop of shear traction on the fault plane. The constitutive law on the fault is assumed to give a stress drop as abrupt as can be resolved numerically. Coulomb yielding off the fault occurs only on the extensional side of the fault. The magnitude of plastic strain is uniform along the fault, and its distribution has a thickness normal to the fault that is proportional to rupture propagation distance.

[71] Energy absorbed off the fault (outside the slip zone) is proportional to the thickness of the plastic strain zone, so it also is proportional to rupture propagation distance. Although this energy is not a constitutive property of a point on the fault, it may be termed fracture energy, because its ratio to available elastic energy released determines rupture velocity. Fracture energy growing in proportion to propagation distance produces constant rupture velocity that is smaller than the Rayleigh speed.

[72] The crack-like solution with fracture energy increasing with propagation distance is self-similar. Solutions at different times are related by scaling length in proportion to time. Velocity and stress at scaled space-time points are invariant. Shear traction on the fault plane drops abruptly, but other components of stress and slip velocity decay more slowly from their peaks as propagation distance increases.

[73] The slow decay of slip velocity is what would be expected from a different rate of drop of shear traction, if the medium were elastic. Using the slip velocity function from the nonelastic calculation as a boundary condition on a fault in an elastic medium, gives stress on the fault in an equivalent elastic problem. The equivalent stress versus slip curves are slip weakening with breakdown slip D_c proportional to propagation distance. Fracture energy on the fault in the equivalent elastic calculation matches that off the fault in the nonelastic calculation.

[74] With energy loss occurring outside the slip zone, fracture energy G and equivalent D_c are not constitutive properties of a location on a fault. Their values at a particular point depend on the dynamic solution that arrives at that point. A yield condition is a constitutive property, and therefore the change of stress components that is allowed by the yield condition for a given initial stress is also a property of a fault location. Peak slip velocity is proportional to peak extensional strain along strike, so it also is a property of a fault location.

[75] A means to imitate off-fault energy loss in a rupture calculation in an elastic medium is to impose an upper limit on slip velocity. The limit on slip velocity is related to the yield condition and to the initial stress state, so it is a property of a location on a fault. For a Coulomb coefficient of friction in the medium of $\tan \phi = 0.75$ and with hydrostatic fluid pressure, the limit on slip velocity is proportional to depth and is 19 m/s at 10 km depth.

[76] Energy loss in the damage zone increases fracture energy in a dynamic rupture above the value relevant to nucleation and above laboratory values. In the nonelastic calculation done here energy loss in the damage zone is a

constant fraction 0.216 of the available elastic energy released. Equivalent D_c is 0.16 of peak slip. These fractions will be smaller at depths greater than 3 km, where confining pressure is larger.

[77] In the conventional paradigm of fracture mechanics, the flow of elastic energy to the rupture front is matched to fracture energy G , which is assumed to be a material constant. This work moves outside that paradigm to make G dependent on the dynamic solution as it arrives at a particular point. In the case calculated here G is a constant fraction of available elastic energy.

5. Conclusions

[78] Energy loss in the damage zone outside the slip zone contributes to the fracture energy that determines the propagation velocity of a rupture.

[79] In a crack-like solution, fracture energy is proportional to propagation distance.

[80] Even with abrupt drop of shear traction in the slip zone, plastic yielding produces a slip-velocity function equivalent to that produced in an elastic medium by a slip-weakening fault friction law with variable breakdown slip D_c , which is proportional to propagation distance in the case of a crack-like solution.

[81] Fracture energy G and equivalent breakdown slip D_c are not constitutive properties of a point on a fault; they depend on the dynamic solution that arrives at that point. Peak slip velocity is proportional to strength of the material, so it is a constitutive property.

[82] Imposing a limit on slip velocity in a calculation is a means to imitate the effect of off-fault energy loss.

Appendix A: Numerical Implementation of Plastic Yielding

[83] The finite difference equations that are used here are given in detail in the appendix of Andrews [1973] for the case of elastic response of the medium. The extension of those equations to plastic response is given here. These equations refer to only a single element, so notation referring to spatial location is omitted. Simplex elements are used, so that stress values are uniform in the interior of an element. The time step number is given as a superscript.

[84] Stress components σ_{xx}^n , σ_{yy}^n , and σ_{xy}^n are known in an element interior at time step n . Velocity components at the nodes of the element at time step $n + 1/2$ are used to find strain rate components $\dot{\epsilon}_{xx}^{n+1/2}$, $\dot{\epsilon}_{yy}^{n+1/2}$, and $\dot{\epsilon}_{xy}^{n+1/2}$ within the element. The procedure to update stress from time step n to $n + 1$ is as follows.

[85] The stress tensor is decomposed into mean stress

$$\sigma_m^n = (\sigma_{xx}^n + \sigma_{yy}^n)/2 \quad (A1)$$

and stress deviators

$$\begin{aligned} s_{xx}^n &= \sigma_{xx}^n - \sigma_m^n \\ s_{yy}^n &= \sigma_{yy}^n - \sigma_m^n \\ s_{xy}^n &= \sigma_{xy}^n \end{aligned} \quad (A2)$$

The dilatation rate $\dot{\Delta}^{n+1/2} = \dot{\epsilon}_{xx}^{n+1/2} + \dot{\epsilon}_{yy}^{n+1/2}$ is used to update mean stress

$$\sigma_m^{n+1} = \sigma_m^n + k \dot{\Delta}^{n+1/2} \Delta t \quad (\text{A3})$$

where k is bulk modulus and Δt is the time step.

[86] Strain rate deviators

$$\begin{aligned} \dot{\epsilon}_{xx}^{n+1/2} &= \dot{\epsilon}_{xx}^{n+1/2} - \dot{\Delta}^{n+1/2} / 2 \\ \dot{\epsilon}_{yy}^{n+1/2} &= \dot{\epsilon}_{yy}^{n+1/2} - \dot{\Delta}^{n+1/2} / 2 \\ \dot{\epsilon}_{xy}^{n+1/2} &= \dot{\epsilon}_{xy}^{n+1/2} \end{aligned} \quad (\text{A4})$$

are used to tentatively update stress deviators elastically

$$\begin{aligned} s_{xx}^{*n+1} &= s_{xx}^n + 2\mu \dot{\epsilon}_{xx}^{n+1/2} \Delta t \\ s_{yy}^{*n+1} &= s_{yy}^n + 2\mu \dot{\epsilon}_{yy}^{n+1/2} \Delta t \\ s_{xy}^{*n+1} &= s_{xy}^n + 2\mu \dot{\epsilon}_{xy}^{n+1/2} \Delta t \end{aligned} \quad (\text{A5})$$

where an asterisk indicates a tentative value and μ is shear modulus. These tentative values are used to evaluate maximum shear stress resolved over all orientations

$$\tau^{*n+1} = \sqrt{(s_{xx}^{*n+1})^2 + [(s_{xx}^{*n+1} - s_{yy}^{*n+1})/2]^2} \quad (\text{A6})$$

which is compared to yield stress Y . If $\tau^* \leq Y$, the material responds elastically, and the tentative values of stress are accepted, $s_{xx}^{n+1} = s_{xx}^{*n+1}$, $s_{yy}^{n+1} = s_{yy}^{*n+1}$, $s_{xy}^{n+1} = s_{xy}^{*n+1}$. If $\tau^* > Y$, the material yields. All stress deviator components are reduced by a common factor, which ensures that plastic strain rate (proportional to the stress adjustment) is in the direction of stress deviators in stress space. If the factor is chosen to be Y/τ^{*n+1} , then stress is adjusted such that $\tau = Y$. Time-dependent relaxation (Maxwellian viscoplasticity) is achieved by letting the reduction factor be $(1 - \exp(-\Delta t/T_v)) (Y/\tau^{*n+1})$, where T_v is the viscoplastic relaxation time. Adjusted stress deviators are

$$\begin{aligned} s_{xx}^{n+1} &= s_{xx}^{*n+1} (Y/\tau^{*n+1}) (1 - \exp(-\Delta t/T_v)) \\ s_{yy}^{n+1} &= s_{yy}^{*n+1} (Y/\tau^{*n+1}) (1 - \exp(-\Delta t/T_v)) \\ s_{xy}^{n+1} &= s_{xy}^{*n+1} (Y/\tau^{*n+1}) (1 - \exp(-\Delta t/T_v)) \end{aligned} \quad (\text{A7})$$

The adjustment to stress $s_{ij}^{*n+1} - s_{ij}^{n+1}$ (stress glut) is an increment to the seismic moment density. Dividing by μ gives the increment of the plastic strain tensor. (Dividing by μ rather than 2μ gives shear strain in the engineering rather than tensor convention. This is done so that plastic strain is volume density of seismic potency.)

$$\begin{aligned} (\Delta \epsilon_{ij}^p)^{n+1/2} &= (s_{ij}^{*n+1} - s_{ij}^{n+1}) / \mu \\ (\epsilon_{ij}^p)^{n+1} &= (\epsilon_{ij}^p)^n + (\Delta \epsilon_{ij}^p)^{n+1/2} \end{aligned} \quad (\text{A8})$$

In stress deviator space plastic strain rate is in the direction of current deviatoric stress (shear stress).

[87] Note that the strain increment determined from the velocities of the nodes of the element is the total strain increment. The adjustment of stress determines what portion of that strain is plastic strain. If the element is unloaded to its initial stress, plastic strain will be the offset from initial strain.

[88] The final step of the procedure is to recompose the total stress tensor

$$\begin{aligned} \sigma_{xx}^{n+1} &= s_{xx}^{n+1} + \sigma_m^{n+1} \\ \sigma_{yy}^{n+1} &= s_{yy}^{n+1} + \sigma_m^{n+1} \\ \sigma_{xy}^{n+1} &= s_{xy}^{n+1} \end{aligned} \quad (\text{A9})$$

[89] If velocity and stress are the fundamental dependent variables that are stored and incremented in the numerical procedure, then nothing more needs to be done to account for the adjustment of stress. If, however, a stiffness matrix is used, so that force and displacement are the fundamental dependent variables, then the adjustment of stress in an element needs to be translated into equivalent body force that is applied to the nodes of the element in all subsequent time steps.

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