

# What Can Strong-Motion Data Tell Us about Slip-Weakening Fault-Friction Laws?

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**Abstract** We consider the resolution of parameters, such as strength excess,  $\sigma^y - \sigma^o$ , and slip-weakening distance,  $d_c$ , related to fault-constitutive properties, that may be obtained from the analysis of strong-ground motions. We show that waveform inversion of a synthetic strong-motion-data set from a hypothetical M 6.5 event resembling the 1979 Imperial Valley earthquake cannot uniquely resolve both strength excess and  $d_c$ . Specifically, we use a new inversion method to find two rupture models, model A having  $d_c = 0.3$  m and high-strength excess, and model B having  $d_c = 1$  m and low-strength excess. Both models have uniform initial stress and the same moment-rate function and rupture time distribution, and they produce essentially indistinguishable ground-motion waveforms in the 0–1.6 Hz frequency band.

These models are indistinguishable because there is a trade-off between strength excess and slip-weakening distance in controlling rupture velocity. However, fracture energy might be relatively stably estimated from waveform inversions. Our Models A and B had very similar fracture energies. If the stress drop is fixed by the slip distribution, the rupture velocity is controlled by fracture energy.

We show that estimates of slip-weakening distance inferred from kinematic inversion models of earthquakes are likely to be biased high due to the effects of spatial and temporal-smoothing constraints applied in such inverse-problem formulations.

Regions of high-strength excess are often used to slow or stop rupture in models of observed earthquakes, but our results indicate that regions of long  $d_c$  and lower strength excess might alternatively explain the slowing of rupture. One way to constrain  $d_c$  would be to model ground-motion spectra at frequencies higher than those at which waveform modeling is possible. A second way to discriminate between regions of long  $d_c$  and large-strength excess might be to assume that  $d_c$  is long where there are no aftershocks.

## Introduction

Two major goals of seismology are to discover the stress conditions on faults before and during earthquakes and to infer a constitutive law that characterizes the material response to the applied stress. A constitutive relation represents the governing equation of the failure process and specifies the dependence between stress, fault slip, slip rate, and other relevant physical properties. While theoretical, numerical, and laboratory simulations of earthquakes have led to tremendous insights into possible constitutive laws, the ultimate test of a constitutive law is its ability to model the behavior of real earthquakes occurring on real faults in the Earth.

The constitutive relation is a key element of dynamic descriptions of the seismic source which are based on models that satisfy the elastodynamics equation (e.g., Andrews, 1976a; 1976b, 1985; Mikumo and Miyatake, 1978; Miya-

take, 1980; Day, 1982; Virieux and Madariaga, 1982; Das and Kostrov, 1987; Harris *et al.*, 1991). In the framework of fracture mechanics, an earthquake may be considered as a dynamically propagating shear crack that radiates seismic waves. The resulting motion (slip history) on the fault is related to a drop in shear stress. The slip evolution depends on the failure criterion, the constitutive properties and the initial conditions on the fault surface. In contrast, kinematic models of the seismic source (e.g., Haskell, 1964) prescribe the displacement history of motion *a priori*, without an explicit attempt to investigate the physical causes of the rupture process.

In some previous works of dynamic simulation, the constitutive relation is assumed to be a simple slip-weakening model (Ida, 1972; Andrews, 1976a; 1976b; Day, 1982) which is completely characterized by initial stress  $\sigma^o$ , yield

stress  $\sigma^y$ , dynamic frictional stress  $\sigma^f$ , and slip-weakening distance  $d_c$  (Fig. 1). In this model the fault begins to rupture when the yield stress,  $\sigma^y$ , is exceeded. As the slip grows to the critical slip-weakening distance,  $d_c$ , the strength of the fault decreases to the dynamic frictional stress,  $\sigma^f$ . The strength excess,  $\sigma^y - \sigma^o$ , is the difference between the yield stress and the initial stress (Boatwright and Quin, 1986). The stress drop is  $\sigma^o - \sigma^f$ .

Strength excess  $\sigma^y - \sigma^o$  has been estimated for many moderate earthquakes by Quin (1990), Miyatake (1992), Fukuyama and Mikumo (1993), Ide and Takeo (1996), Beroza and Mikumo (1996), Bouchon (1997), Day *et al.* (1998), and others. Of particular interest are two articles that also estimated slip-weakening distance  $d_c$ . The first is Ide and Takeo (1997), who derived a constitutive relation from inversion of strong-ground-motion data from the 1995 Hyogo-ken Nanbu (Kobe), Japan, earthquake. They inferred a  $d_c \approx 1$  m for the shallow part of the fault, and they estimated an upper bound of 0.5 m for  $d_c$  on the deeper part of the fault. The second article is by Olsen *et al.* (1997), who estimated  $d_c = 0.8$  m for the 1992 Landers, California, earthquake by dynamically simulating previously obtained kinematic models of the Landers event. These estimates of  $d_c$  are quite high compared to values measured in the laboratory, typically of the order of 10  $\mu$ m (e.g., Dieterich, 1978). While there are reasons to expect that  $d_c$  on real faults might be much larger than in the laboratory (Scholz, 1988; Okubo and Dieterich, 1984; 1986), it is also important to assess the reliability of the estimates of  $d_c$  derived from strong-motion data.

The primary purpose of this article is to answer the following questions. (1) Can the slip weakening distance and strength excess be uniquely determined by modeling ground motion data? (2) Can the slip-weakening distance and the strength excess be inferred uniquely from a kinematic source model? and (3) How wide a range of these parameters is consistent with a given ground-motion-data set?

Of course, although slip-weakening distance is probably a material property, strength excess and stress drop are not material properties because they are defined with respect to some unknown initial stress level. If slip-weakening is the constitutive law that governs real-fault friction, then the parameters that yield stress  $\sigma^y$  and dynamic-frictional stress  $\sigma^f$  would be material properties. However, lab experiments on homogeneous rock samples (Okubo and Dieterich, 1986) show that even yield stress and dynamic friction stress are not material properties because they vary from place to place on the homogeneous rock samples.

More complicated fault-constitutive laws are required to explain some aspects of earthquake behavior, namely, the restrengthening of faults between earthquakes. However, results of laboratory experiments on rock-on-rock frictional sliding at high-speed slip (Okubo and Dieterich, 1986) and of numerical modeling of dynamic rupture with laboratory-derived constitutive laws (Okubo, 1989) suggest that slip-weakening is the prevailing constitutive behavior during dynamic rupture. Therefore, in this article, we adopt slip-

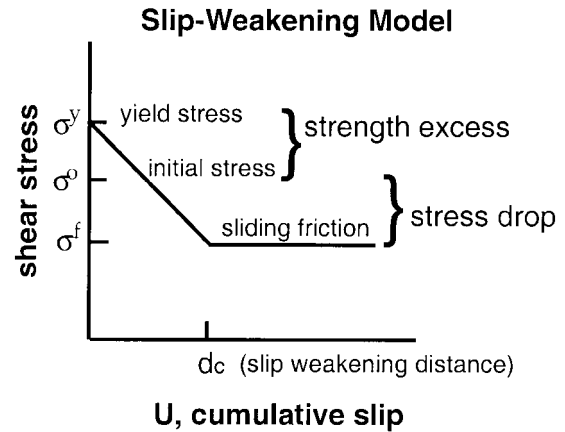


Figure 1. Slip-weakening friction law (Ida, 1972; Day, 1982; Andrews, 1985). The curve represents the total shear stress as a function of cumulative slip.

weakening as a relatively simple model for which we can try to recover the defining parameters from observations of earthquakes.

In this article, we present an inversion method for deriving dynamic rupture models that satisfy both a given kinematic source model and constraints on desired properties of fault-constitutive parameters. This inversion method can be generalized to invert seismograms. We use this procedure to demonstrate that there is a strong trade-off between the strength excess and the slip-weakening distance in controlling rupture velocity. We find that many different combinations of these two parameters cause the same rupture time distribution, and consequently radiate similar ground motions. We show that this trade-off might be equivalent to considering that, if the stress drop is fixed by the slip distribution, a unique constitutive parameter, the apparent fracture energy, controls the rupture velocity. We show that waveform inversion of a synthetic strong-motion-data set from a hypothetical *M* 6.5 event in the frequency band 0–1.6 Hz cannot uniquely define stress parameters and slip weakening distance  $d_c$ . Specifically, we use our inversion method to produce two rupture models having a  $d_c = 0.3$  m and  $d_c = 1.0$  m, and we find that these two rupture models produce indistinguishable ground-motion waveforms in the 0–1.6 Hz band. Moreover, these rupture models have identical rupture times and very similar slip distributions and rise times (the time required to accumulate 10 to 90% of the total slip at individual points). Given these difficulties, it may be necessary to resort to broadband spectral domain studies in order to define constitutive relations for moderate-sized earthquakes.

We will demonstrate our results using theoretical seismograms from a hypothetical test earthquake designed to resemble the *M* 6.5 1979 Imperial Valley, California, earthquake. Our initial goal had been to apply the new inversion method to the observed ground-motion data from the Imperial Valley earthquake in order to determine the constitu-

tive law. In the process of preparing synthetic test rupture models in order to test the inversion method, we discovered the nonuniquenesses that we present in this article.

### An Ideal Test Case

Our approach is to show that, in general, the relevant kinematic properties of a source, such as rupture velocity, final slip, and moment-rate function are consistent with a broad range of slip-weakening constitutive laws. In general, slow rupture velocity is caused by either a long slip-weakening distance or a high strength excess, and the opposite is true for a fast rupture propagation. A simple way to state this type of ambiguity is that a high strength–short slip-weakening distance model is equivalent to a low strength–long slip-weakening distance model. We have generated two dynamic models that represent these two extremes and conform equally to the important kinematic parameters. The first step to accomplish this task was the design, by means of a spontaneous rupture calculation, of a reference model whose dynamic properties lie between those two extreme characterizations. The resulting rupture velocity, moment rate function and final slip represent the reference kinematic properties which the two equivalent dynamic models must both satisfy.

Ideally, an earthquake that has relatively long rise times (the duration of slip at individual points) and ground-motion data that can be modeled accurately at high frequencies would present the best opportunity to resolve details of the rupture process. In order to decide which earthquake we should try to simulate, we surveyed published waveform inversion studies. We visually examined the waveform fits of Hartzell and Heaton (1983), Archuleta (1984), Hartzell (1989), Beroza (1991), Steidl *et al.* (1991), Wald *et al.* (1991), Fukuyama and Mikumo (1993), Cohee and Beroza (1994), Wald and Heaton (1994), Cotton and Campillo (1995), Wald (1996), Ide *et al.* (1996), and Ide and Takeo (1997), and we determined that for all of these, with the possible exceptions of Ide *et al.* (1996) and Ide and Takeo (1997), the shortest modeled period was equal or longer than the main asperity rise time in the published kinematic model, meaning that these studies cannot be used to infer fault-constitutive relations. In the latter two papers on the Kobe earthquake, almost all the modeled data were long period, but two short period stations were modeled to periods of about 2 sec. Cotton and Campillo (1995) showed spectra of their waveform misfits, a laudable practice that should be encouraged in future inversions.

We chose to simulate an earthquake that resembles the 1979 Imperial Valley earthquake because that event offers possibly the best data set for determination of constitutive relations. In fact, Hartzell and Heaton (1983) obtained a good waveform fit at a 1.5-sec period at several stations, and they and Archuleta (1984) found a rise time of 1.7 and 1.8 sec, respectively, on the main asperity. Given the excellent station geometry (Fig. 2) and the approximately laterally ho-

### IMPERIAL VALLEY STATIONS AND FAULTS + Archuleta (1982) hypocenter

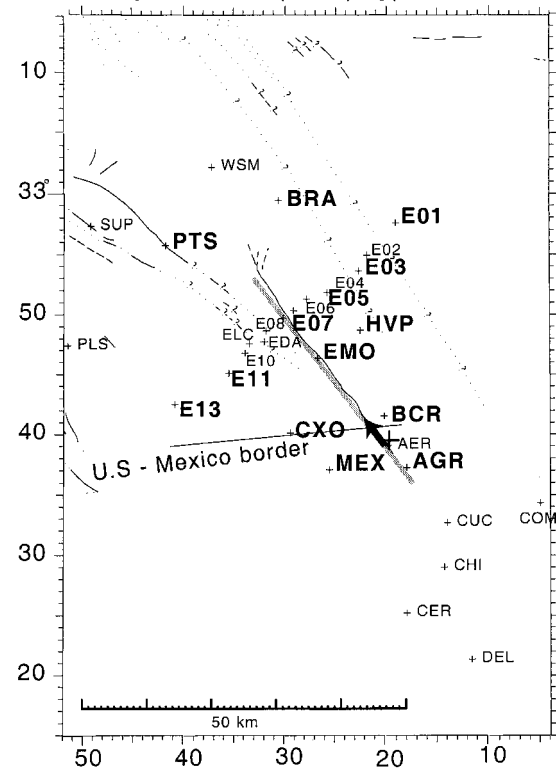


Figure 2. Map showing surface trace of the Imperial Fault and the stations that recorded the 1979 earthquake. In this study we have calculated synthetic seismograms at the stations shown in bold characters. The thick line corresponds to the fault plane used in our modeling, and the arrow shows the direction of rupture propagation. Cross is the hypocenter.

mogeneous local-velocity structure (Fuis *et al.*, 1984), there is hope that its waveforms could be modeled to periods of 1 sec or shorter at many stations.

Figure 2 shows the map view of the Imperial Valley area with the station distribution, the fault trace, and the epicenter location. The Imperial Valley earthquake had substantial surface slip (Sharp *et al.*, 1982) and a well-determined epicenter (Archuleta, 1982), thus giving a fault length of about 35 km northwest of the epicenter (Archuleta, 1984). The inversion results of Olson and Apsel (1982) indicate that slip occurred also on a 10-km segment south of the epicenter.

For the parameterization of our models, we assume a fault plane that extends about 33 km north of the epicenter and 8 km south of the epicenter along a strike of  $323^\circ$ , and reaches a depth of 14 km with a  $80^\circ$  NE dip. We used the hypocenter of Archuleta (1982) at depth of 8 km. We gridded the fault plane into 988 subfaults (52 along strike and 19 along dip) each about 0.8 km square.

The first step in our investigation was the creation of a reference dynamic rupture model. Specifically, on our simulated Imperial Fault we had to find distributions of strength

excess, stress drop, and slip-weakening distance, which would lead to a rupture that had approximately the same slip and rupture-time distributions as those determined by Archuleta (1984) for the 1979 Imperial Valley earthquake. This task was accomplished by trial-and-error forward modeling using the boundary integral spontaneous rupture computer code developed by Boatwright and Quin (1986), Quin and Das (1989), Quin (1990), and modified by Spudich *et al.* (1998) to include free-surface reflections and vertical variations of the velocity structure.

We assumed a uniform initial shear stress field and we applied the method of Spudich *et al.* (1998) to derive the stress drop  $\sigma^o - \sigma^f$  distribution from the slip distribution of Archuleta's model (1984), and by trial and error we found that a spatially uniform strength excess of about 4 MPa and a uniform slip-weakening distance of about 0.5 m yielded slip and rupture-time distributions similar to Archuleta's kinematic model. Our reference model lacked the super-shear rupture velocity observed by Archuleta, but this discrepancy was irrelevant to our subsequent use of the reference model.

Then we used the inversion method described in the Appendix to generate two rupture models—one characterized by a short slip-weakening distance ( $\sim 0.3$  m) and high-strength excess (model A), and the other by a long slip-weakening distance ( $\sim 1$  m) and low-strength excess (model B), both of which are consistent with the moment rate function, rupture time, slip, and initial shear-stress distributions of the reference model. Although these values of slip-weakening distance are large compared to the values implied by results of rock friction experiments, the scaling of this constitutive parameter for real faults during dynamic rupture is still not known, and recent proposed estimates based on waveform inversion are of the order of 0.5 m (Ide and Takeo, 1997; Olsen *et al.*, 1997). In general, using waveform inversion, we can only resolve these very large slip-weakening distances from bandlimited data. Then, if we cannot distinguish a slip-weakening distance of the order of 0.3 m from one of about 1 m, clearly slip-weakening distances shorter than these values cannot be resolved from narrowband data.

A uniform initial shear stress might not be a realistic initial condition for a dynamic representation of a real event (e.g. Spudich *et al.*, 1998; Olsen *et al.*, 1997). Within this contest, Olsen *et al.*, (1997) proposed a dynamic model for the 1992 Landers earthquake in terms of true constitutive parameters of the fault (in the framework of a slip-weakening model), by assuming a nonuniform initial shear stress and uniform material properties (peak stress, frictional stress, and slip-weakening distance). Of course, a general characterization of seismic source should account for heterogeneities in both the initial stress conditions and in the constitutive properties.

## Results

In this section we analyze the characteristics of the two dynamic models A and B obtained with the inversion de-

scribed in the Appendix. These models are kinematically equivalent because both have the same moment-rate function, slip distribution, and rupture time, but they differ in the two critical parameters of the fault-constitutive relation, the strength-excess distribution and the slip-weakening distance. We shall show later that their ground motion waveforms are very similar.

Figure 3a shows the rupture-time distribution common to both models imposed through constraint K2 (Appendix), and the similarity of the moment rate-functions is evident in Figure 3b. Figure 4 shows all the relevant dynamic and kinematic parameters for the two models. Note that the large values of stress drop (and fracture energy) on the left side of Figures 4c and 4d are only due to an edge effect in the numerical computation. The final slip distribution (Fig. 4a,b) is remarkably similar—model B shows larger slip maxima than model A—but these differences would probably be too small to be resolved by either geodetic observations or low frequency waveform data, given that both models basically satisfy the same moment rate function (Fig. 3b). The static stress drop distributions (Fig. 4c,d) are bounded by the static-stress-drop distribution of the reference model (constraint D1, Appendix). They have small differences caused by the different scaling used in the two cases.

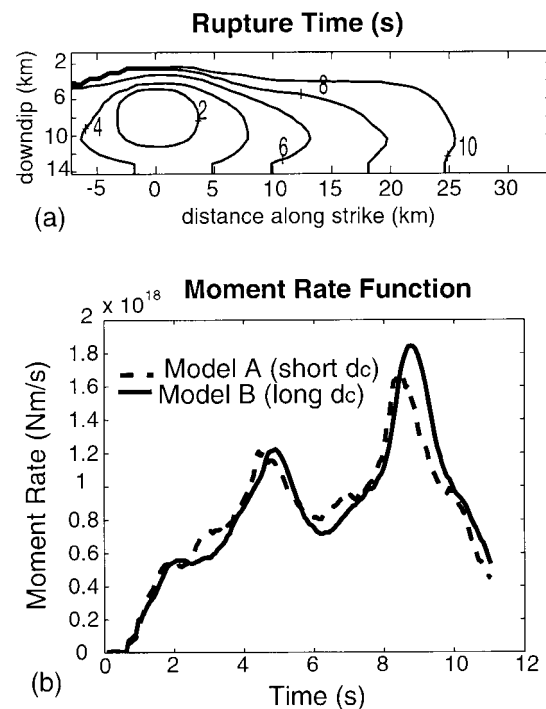


Figure 3. (a) Rupture time distribution for both model A and model B. We impose the rupture-time distribution of the reference model on both models (constraint K2). (b) Moment rate function for model A (dashed line) and model B (continuous line). We force both models to be consistent with the moment rate function of the reference model (constraint K1).

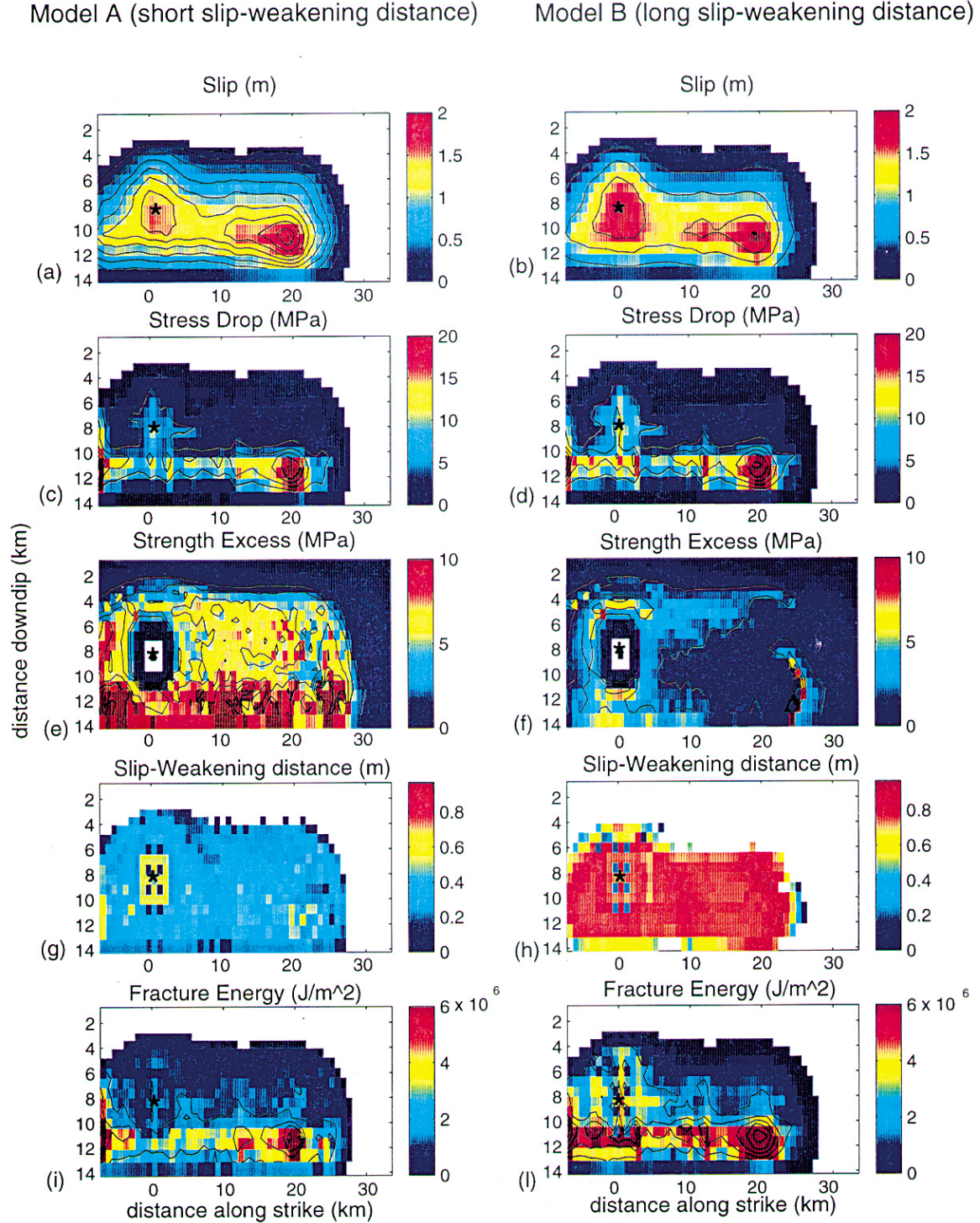


Figure 4. Distribution of kinematic and dynamic parameters on the fault plane (the asterisk indicates the hypocenter location) for model A (left) and model B (right). Slip distribution (a) and (b); stress drop distribution (c) and (d); strength excess distribution (e) and (f); slip-weakening distance distribution (g) and (h); apparent fracture energy distribution (i) and (j). Note that the two models have very similar slip and stress-drop distributions, as required by the constraints imposed, but they have very different strength excess and slip-weakening distance distributions, due to the existing trade-off between these two parameters in controlling the rupture velocity. The similarity between the apparent fracture energy distribution for the two models suggests that this parameter might be relatively stably estimated from waveform inversions.

### Trade-Off between Strength Excess and Slip-Weakening Distance

The strength excess (Fig. 4e, f) and the slip-weakening distance distributions (Fig. 4g, h) differ substantially, and their analysis is important to understand the origin of non-uniqueness in the problem of fitting kinematic properties with spontaneous rupture modeling. Model A has a slip-weakening distance of the order of 0.3 m (as required by constraint D3, Appendix), and for model B this constraint results in a slip-weakening distance of the order of 1 m; on the other hand the strength excess in model A is substantially larger than that in model B. It is important to note that the strength excess is not directly constrained by any imposed requirement; at each point it is simply the stress at the instant that point is allowed to rupture (Miyatake, 1992). In Figure 5 we depict the slip-weakening curves for selected subfaults over different depths and distances along the strike of the faults. From this figure it is possible to appreciate the differences in the rate of weakening as imposed by constraint D3.

Let us consider the strength parameter  $S = \frac{\sigma^y - \sigma^o}{\sigma^o - \sigma^f}$  and its role in controlling the rupture velocity. Andrews (1976b; 1985) showed that for  $S$  values greater than 1.77, the limiting rupture velocity in plane strain is the Rayleigh velocity. If  $S$  is less than 1.77, however, when the crack has propagated a sufficient distance, the rupture velocity can become larger than the  $S$ -wave velocity (Andrews, 1976b; Das and Aki, 1977; Day, 1982). In a slip-weakening model the transition in rupture velocity depends on the half-length of a critical crack  $L_c$  (equation 10 in Day, 1982 and equation 43 in Andrews, 1985), and therefore on  $d_c$ . This means that

in these studies the separate effect of  $d_c$  on the rupture velocity was implicitly taken into account through the length scale  $L_c$ . Our simulations show that, in a spontaneous rupture calculation of a finite source, a subshear rupture velocity can be equally attained by combining a relatively low  $S$  parameter and a large slip-weakening distance, or a high  $S$  with a short slip-weakening distance. If the stress drop is fixed, then the relevant parameters that trade off are the strength excess and the slip-weakening distance

### Apparent Fracture Energy

Figures 4i and 1 show the distribution of the apparent fracture energy  $G_c$  for the two models. The shear-fracture energy is defined as the amount of energy needed to create a unit area of fault by shear fracture, and in a simple slip-weakening model is calculated as

$$G_c = \frac{1}{2} (\sigma^y - \sigma^f) d_c \quad (1)$$

(Andrews, 1976a).

Estimates of shear-fracture energy have been calculated for many earthquakes (e.g., Ida, 1973; Hussein *et al.*, 1975; Aki, 1979; Beroza and Spudich, 1988), yielding a range from  $10^2$  to  $10^8$  J/m<sup>2</sup>. Our models yield an apparent fracture energy of the order of  $10^6$  J/m<sup>2</sup>, and this value may provide a rough estimate of this parameter for a moderate-size event such as the 1979 Imperial Valley earthquake. The distributions of  $G_c$  for the two models are remarkably similar and resemble the respective stress-drop distributions, suggesting that the apparent fracture energy might be stably estimated from waveform inversions.

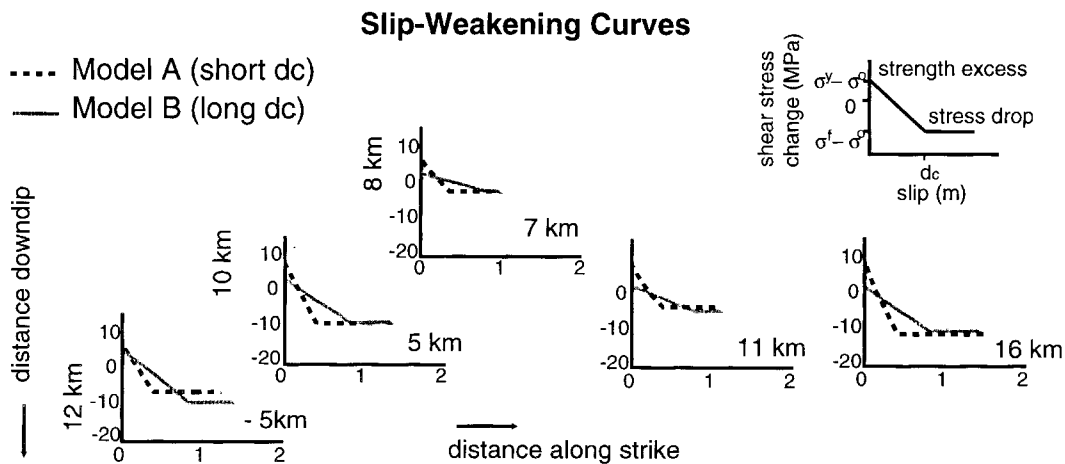


Figure 5. Slip-weakening curves at selected subfaults for model A (dashed line) and model B (continuous line) at different position along strike and different depths. We impose a fast weakening to model A (resulting in a slip-weakening distance of about 0.3 m) and a slow weakening to model B (resulting in a slip-weakening distance of about 1 m). Note that for model A the strength excess is systematically larger than that for model B.

### Radiated Waveforms and Spectra

We compare the seismograms and the spectra calculated from model A and model B up to  $\sim 2$  Hz at 14 observer locations well distributed around the rupture area of the 1979 Imperial Valley earthquake (Fig. 2). We first calculated synthetic ground motions for both a right-lateral, strike-slip dislocation and a normal (east side down) dislocation of constant amplitude over each of the subfaults and for each observer. These seismograms were calculated using the methods of Olson *et al.* (1984) and Spudich and Archuleta (1987) assuming the velocity structure of Archuleta (1984). We calculated the seismograms in the frequency band 0–1.8 Hz and we lowpass filtered them with a cosine taper between 1 and 1.8 Hz. The waveforms radiated by model A and model B are calculated by properly summing over the time steps the contributions of each subfault synthetic weighted by the respective slip increment.

Figure 6 shows the seismograms calculated from model A and model B. In general they are nearly identical, especially for the low frequency part of the signals. This similarity shows that it will be very difficult to resolve constitutive parameters from waveform inversion in this frequency band. A detailed analysis of the waveforms reveals some differences. In general, the seismograms from model A are richer in high frequency than those from model B. This enhanced short-period content is particularly evident for stations close to the fault (e.g., EMO) that are in the forward directivity region (BRA and PTS), and at stations like AGR and MEX which fall in the backward directivity zone where the details of the rupture process (close to the hypocenter) are better resolved.

For waveform inversions, these distinctions would not be sufficient to discriminate the details of the rupture process, mainly due to the inherent difficulty of modeling high frequency signals given our usually inadequate Green's functions. The differences between the model A and model B seismograms are substantially smaller than the misfit usually accepted in modeling real waveform data. However, this analysis suggests that the frequency content of acceleration data might help bound the parameters of interest of fault constitutive behavior.

In Figure 7 we show the displacement-amplitude spectra for the three components of ground motion at some stations in different positions around the fault. The differences in frequency content and relative amplitude already discussed in the seismogram analysis are especially evident in the spectra. Above  $\sim 1$  Hz the amplitude spectra from model A are larger than those from model B, especially at stations close to the fault (e.g., EMO and AGR), in the forward-directivity region (BRA), and in the backward directivity region (AGR and MEX). However, neither the spectra nor the waveforms for stations distant from the fault (e.g., E05 and E11) show systematic differences.

Figure 8 summarizes the spectral content of the three components of motion for all 14 stations used in this study.

For each component we show the ratio of the smoothed amplitude spectra of model A and that of model B; the thick line represents the mean of these ratios, and the dashed lines correspond to the mean  $\pm$  the standard deviation. For the vertical component, the differences in the spectral amplitudes between the two models are negligible, as shown by the ratio of amplitude spectra having a mean value of about 1. On the other hand, for the two horizontal components there is a systematic increase in the ratio of amplitude spectra above 1 Hz, reaching a mean value of about 2.5 at 1.6 Hz. This analysis demonstrates that the two dynamic models are equivalent from the point of view of fitting the low frequency content of the observational constraints, while the specific characteristics that distinguish them affect only the short period part of the data.

In the following section we analyze the resulting discrepancies in the source-time functions in order to explain, in term of the physics, the differences noted in the waveforms and in the spectra.

### Source Time Function and Rise Time

The analysis of the resulting source-time functions (STFs) highlights that the two source dynamic descriptions, although consistent with the kinematic constraints K1 (moment rate) and K2 (rupture time), differ in other, more detailed, kinematic parameters that mostly affect the short-period content of ground-motions data. The features that characterize and distinguish the source-time functions of the two models (Fig. 9) can be generalized as follows: for model A the source-time function follows a typical Kostrov-like functional form with large peak-slip velocity (and fast decay); for model B the source-time function shows a typical triangular shape (or an overlapping of different triangular functions) with lower peak-slip velocity than for model A. The large peak-slip velocity in model A is related to a large-peak stress (strength excess) as shown by Andrews (1976a) and Ohnaka and Yamashita (1989). At every subfault the total slip implied by each model is about the same, meaning that the zero frequency part of the data is not affected by these differences.

The STF for model B can be interpreted as a low-passed version of the STF for model A, therefore, the ground motions calculated from the two models are equivalent for the low frequency part of the signal (in this case up to  $\sim 1$  Hz). At higher frequencies, the differences in the details of the rupture process will affect the seismograms.

The differences in the shapes of the STF resulting from the two models suggest that there might be a specific time parameter that distinguishes the two cases. The rise times (here defined as the time required to accumulate from 10% to 90% of the total slip) for both models are very similar (Fig. 10). The similarities between the two distributions imply that the analysis of this parameter (as it is defined here) would not help resolve the dynamic nonuniqueness discussed in this study, at least for differences in the value of

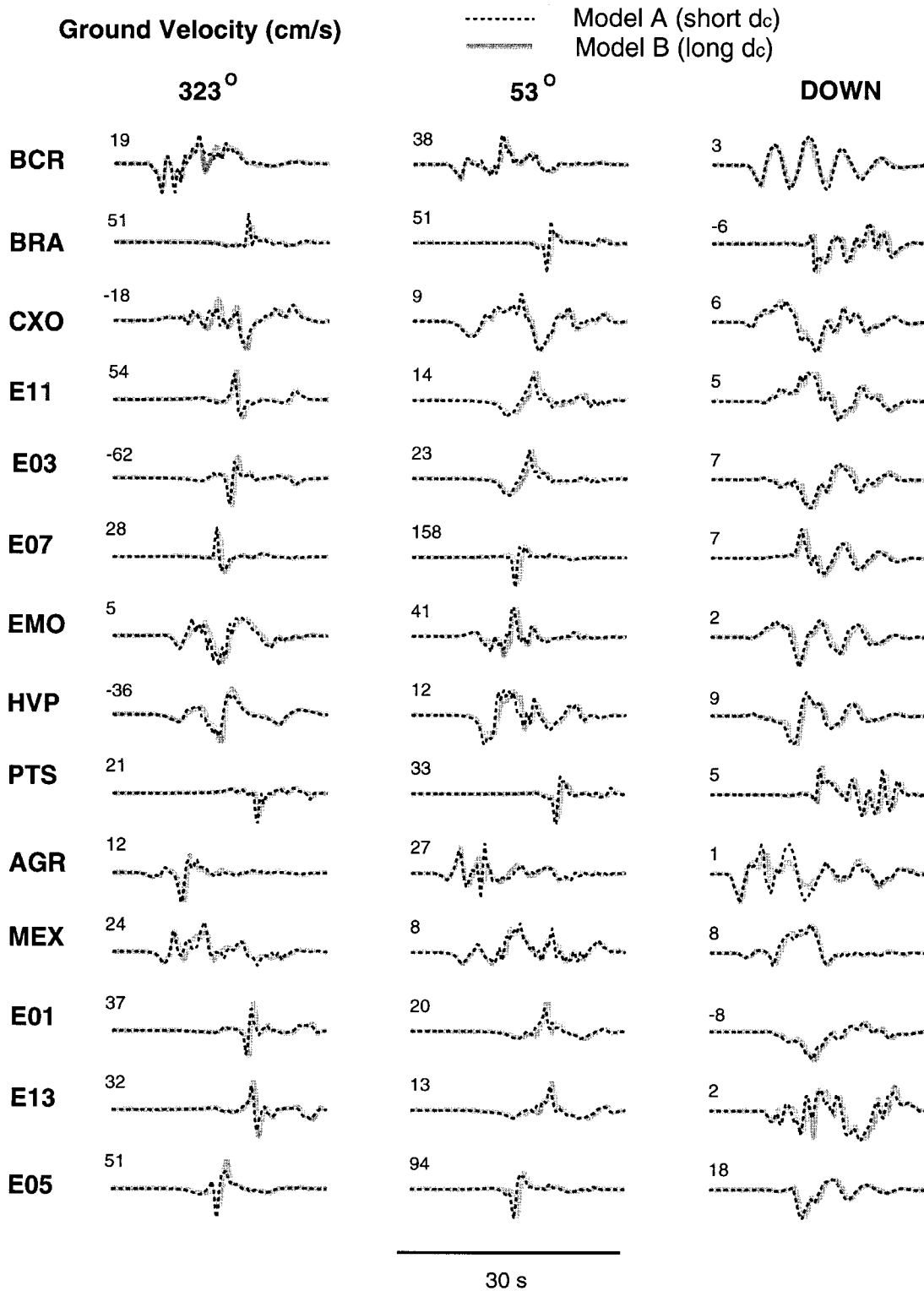


Figure 6. Synthetic seismograms resulting from model A (dashed line) and model B (continuous line). We show the three components of ground velocity (cm/sec) calculated at the stations shown in Figure 2; the number on the upper left corner of each trace indicates the respective maximum amplitude. The synthetics are calculated up to 1.8 Hz and then lowpassed with a cosine taper between 1 and 1.8 Hz. The similarity between the Model A and B waveforms shows that it would be very difficult to distinguish between these models by waveform inversion.

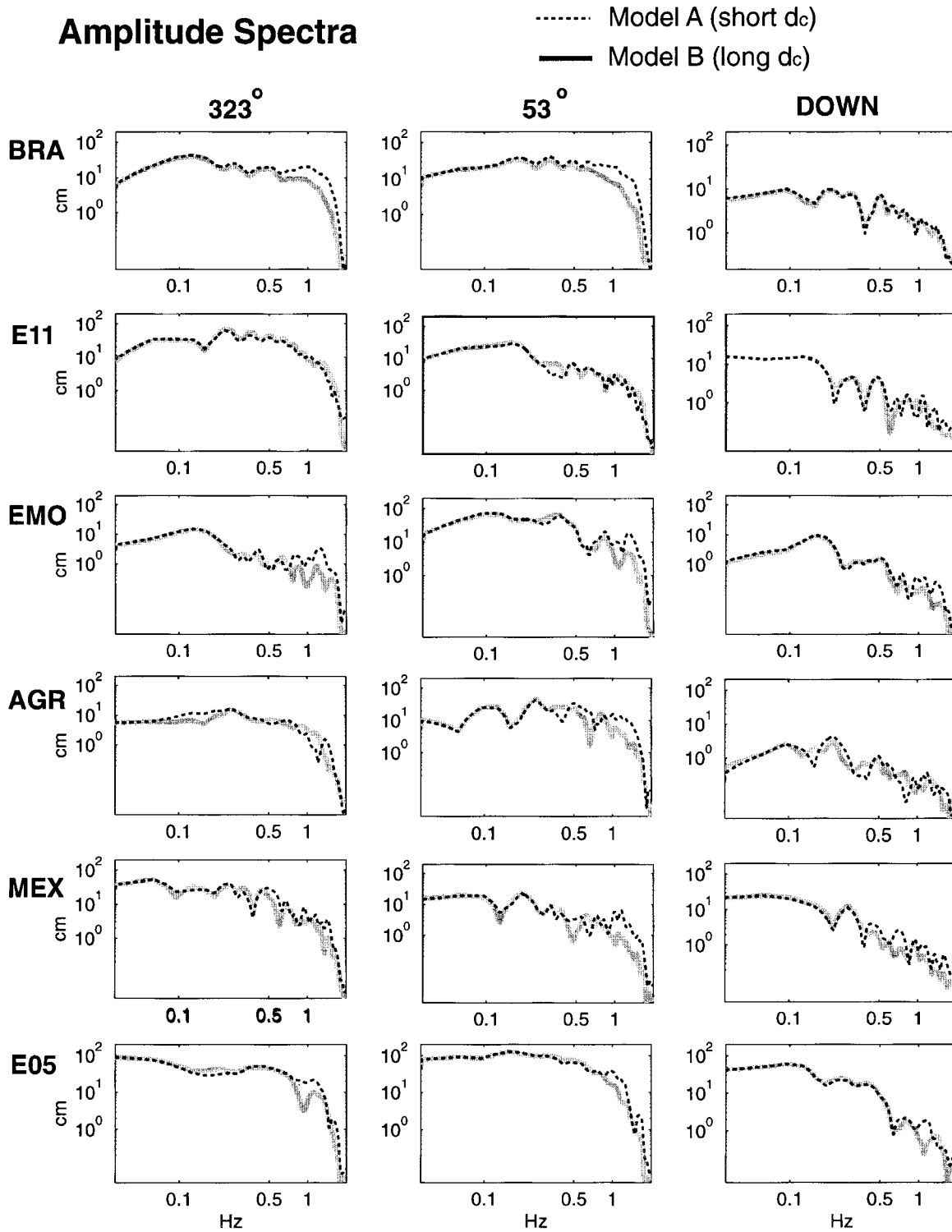


Figure 7. Amplitude spectra resulting from model A (dashed line) and model B (continuous line) for selected stations. In general, model A radiates more energy at frequencies above about 1 Hz than model B. This is particularly evident at stations in the forward directivity region (BRA), close to the fault (EMO), and in the backward directivity region (AGR and MEX), and the differences between the two models are almost negligible at stations more distant from the fault (E11). These amplitude spectra are derived from synthetics calculated up to 1.8 Hz and then lowpassed with a cosine taper between 1 and 1.8 Hz.

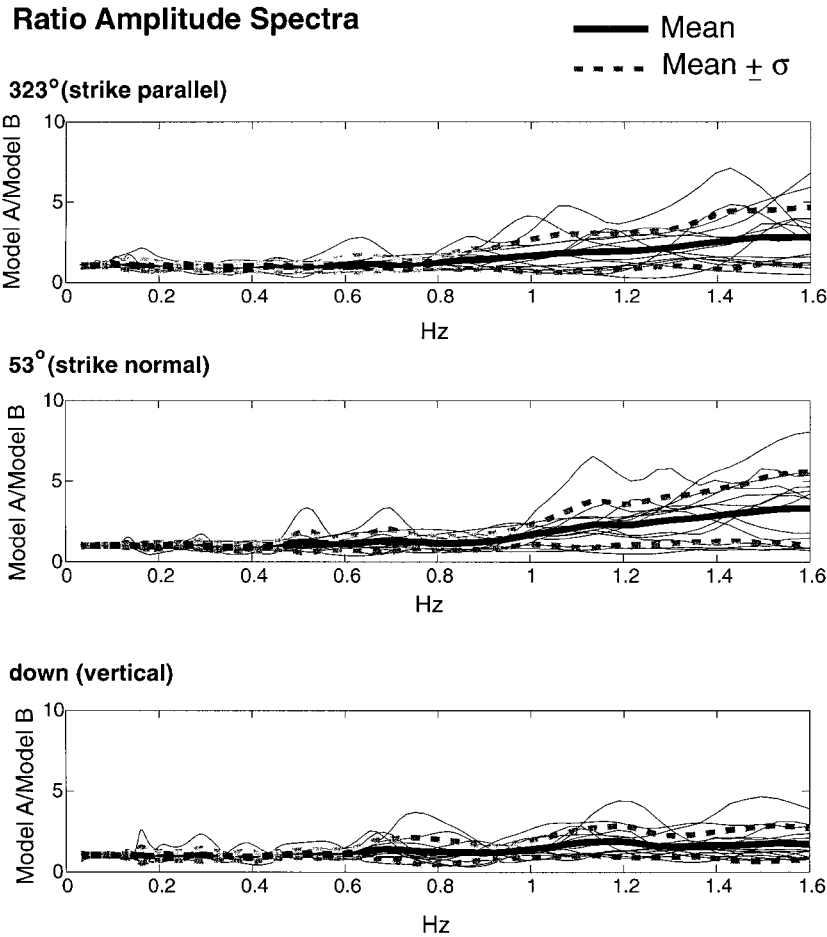


Figure 8. Ratio between the smoothed amplitude spectra of model A and those of model B. For each component of motion we show the amplitude spectra ratio calculated at each station (thin lines) and the mean calculated among all the observations (thick line). The thick dashed lines are the mean  $\pm$  the standard deviation. For the two horizontal components the mean of the amplitude spectra ratio is above 1 for frequencies larger than about 1 Hz, reaching a value of about 2.5 at 1.6 Hz. This means that model A radiates more energy above about 1 Hz than model B, and that the two models are equivalent at low frequencies, and the differences in the details of the rupture models are evident only at high frequency.

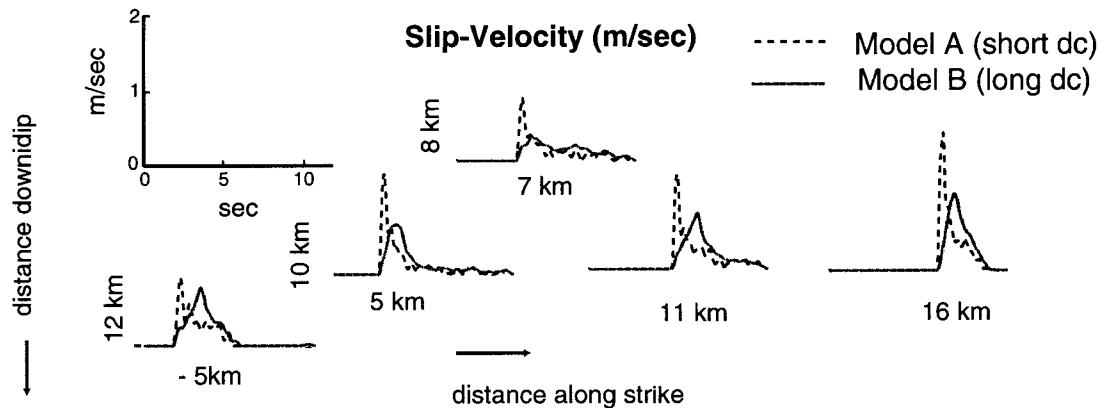


Figure 9. Slip-velocity histories (or source-time functions) for model A (dashed line) and model B (continuous line) at different positions along strike and at different depths as in Figure 5. The source time functions for model A have larger peak-slip velocity, faster decay, and the same total slip as those of model B, for which the slip velocity follows a typical triangular shape (or an overlapping of different triangular functions).

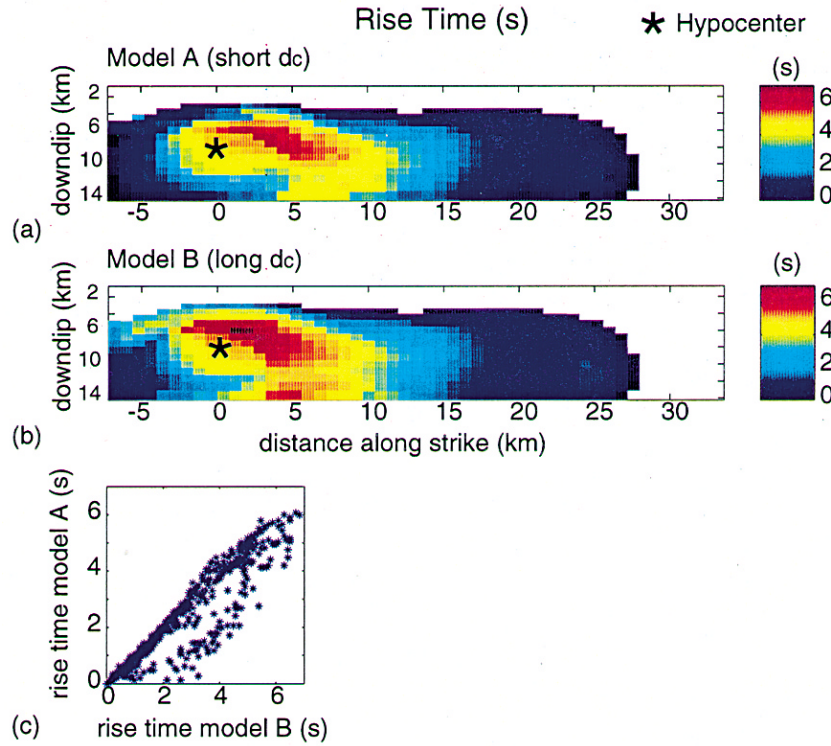


Figure 10. Comparison of rise time distribution (calculated as the time required to cumulate 10 to 90% of the final slip) between model A (a) and model B (b). The similarities between the two distributions imply that the analysis of this parameter would not help distinguish the two models. (c) Rise time of model A plotted as a function of rise time of model B.

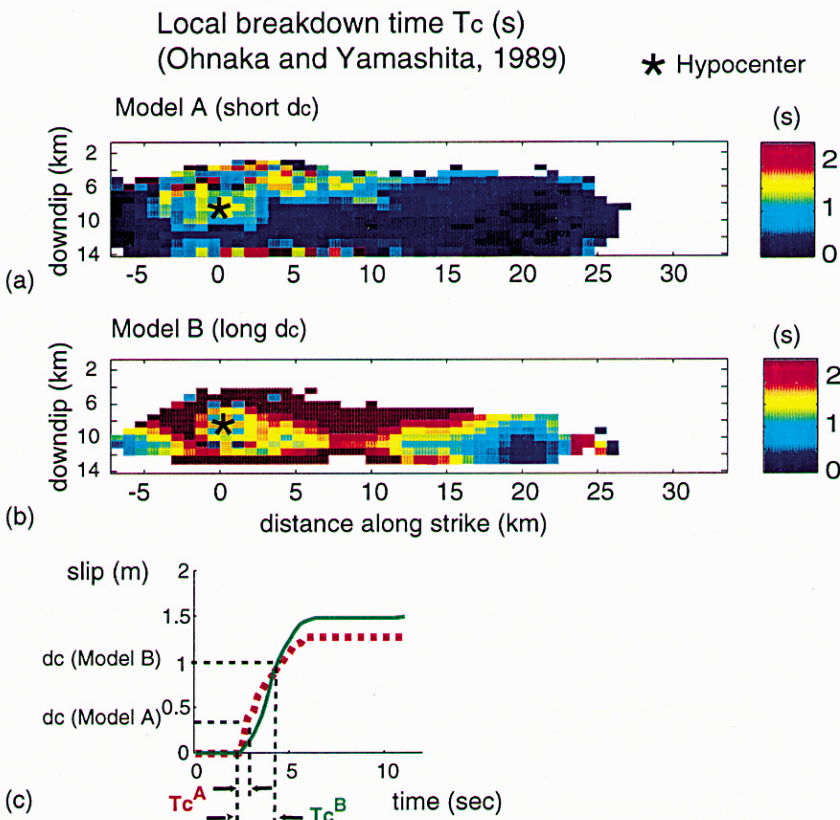


Figure 11. Comparison of local breakdown time distribution (defined as the time over which the weakening occurs) between model A (a) and model B (b). The local breakdown time for model A is systematically shorter than that in model B, as we should expect from the difference in the rate of weakening characteristic of the two models. (c) Local breakdown time for the two models relative to the respective slip histories.

slip-weakening distances such as those considered in this article.

#### Local Breakdown Time and Slip-Weakening Distance Resolution

An important time parameter that characterizes and distinguishes the two dynamic models is the local breakdown time  $T_c$ , defined as the time over which the weakening occurs (Ohnaka and Yamashita, 1989) (Fig. 11). In a slip-weakening model, this is the shortest period that characterizes the rupture process. Because the generation of high-frequency radiation is directly related to details of the rupture process, the larger generation of short-period radiation of model A compared to model B is caused by the rate of weakening (and therefore the breakdown time). From Figure 11 we observe that  $T_c$  for model A is much shorter than that of model B over the fault plane, this short breakdown time being caused by both a short slip-weakening distance and a high-strength excess (Figs. 5 and 11). The differences in the rate of weakening between model A and model B are evident in the rate of decay of their respective slip-velocity histories.

The local breakdown time is the period that we must resolve if we aim to learn something about the weakening process. The shortest modeled period in the ground-motion data is the shortest resolvable period in the rupture process,  $T_{cmin}$ . From  $T_{cmin}$  it is possible to estimate the shortest resolvable slip-weakening distance  $d_{cmin}$  from a given data set. At a given point on the fault, let  $T_c$  be the true breakdown time and  $\bar{s}$  be the average slip velocity over  $T_c$ ; then the true slip-weakening distance  $d_c$  is  $d_c = \bar{s}T_c$ . If we can roughly estimate  $\bar{s}$  from ground motions data, then

$$d_{cmin} = \bar{s}T_{cmin} \quad (2)$$

This relation is plotted in Figure 12 for different values of  $\bar{s}$  and gives a rough idea of the level of resolution of slip-weakening distance based on the maximum frequency in the data.

Based on the above results, it appears that the estimate of the slip-weakening distance from a kinematic source model might be influenced by the specific source time function chosen to parameterize the time dependence of the slip.

### Discussion

#### Interpretation of Previous Estimates of Slip-Weakening Distance and Strength Excess

Stress drop and rupture velocity are relatively well determined by fitting narrowband waveforms. On the other hand, the slip-weakening distance is not uniquely determined because in a dynamic slip-weakening model, resistance to rupture is obtained either from large-strength excess or large slip-weakening distance.

Regions of high-strength excess are often used to slow or stop rupture in modeling observed earthquakes. For ex-

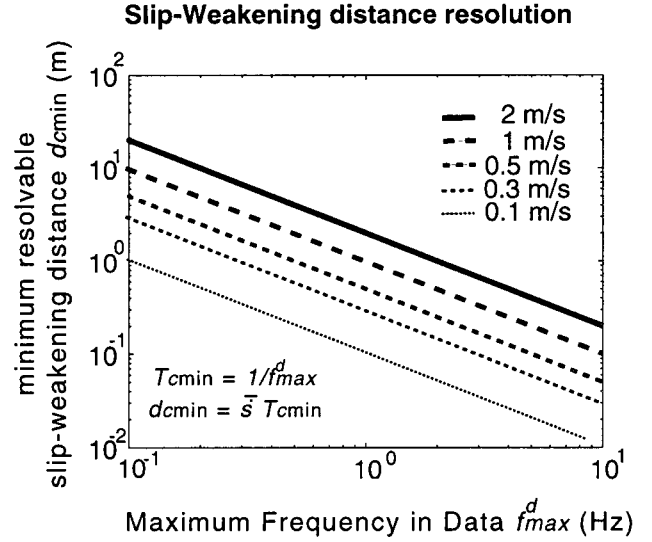


Figure 12. Slip-weakening distance resolution as a function of the maximum frequency contained in the data used for inferring a source model. The different lines correspond to different values of average slip-velocity in the early part of slip as representative of average slip-velocity during weakening.

ample, the numerical methods of Fukuyama and Mikumo (1993), Mikumo and Miyatake (1993), Beroza and Mikumo (1996), and Ide and Takeo (1996) all use a critical-stress-fracture criterion, in which there is no slip-weakening distance, so only a high-strength excess can be used to stop rupture in these models. The levels of strength excess appearing in their models are upper limits on the values consistent with the data. Because the fracture energy in these models of critical-stress-fracture criterion is ambiguous, we cannot estimate plausible combinations of strength excess and nonzero slip-weakening distances for their models.

Our results suggest that slip-weakening distances inferred from kinematic-inversion models of earthquakes are biased high because such inversions are typically constrained to have a fairly smooth slip-rate functions. Clearly, a slip-rate function that initiates with a Kostrov-like  $t^{-1/2}$  singularity will have a slip-weakening distance of zero. Any slip-rate function initiating more gradually will have a longer slip-weakening distance. Slip-rate functions consisting of overlapping triangular (e.g., Wald, 1996; Ide *et al.*, 1996) or boxcar functions (Yoshida *et al.*, 1996) necessitate a slow-weakening behavior resulting in relatively long slip-weakening distance compared with the final slip. In addition, the spatial and temporal smoothing constraints applied in such inverse problem formulations average the slip-velocity, lowering possible high initial peak-slip velocities typical of fast weakening. These effects combine and bias the estimate of the slip-weakening distance to one extreme (i.e., to large values) of the range of possible solutions consistent with the data. Moreover, from relation (2) and Figure 12 it appears that the limited temporal resolution imposed by the shortest

modeled period in the ground motion introduces another type of systematic error in  $d_c$  distributions inferred from inversions of narrowband strong-motion data. This systematic error causes  $d_c$  to be correlated with the average slip-velocity and possibly with the total slip in each subfault.

The use of triangular slip velocity functions and of temporal smoothing probably explains the inability of Ide and Takeo (1997) to resolve slip-weakening distances less than 0.5 m. However, as they emphasize, the inferred differences in the rate of weakening between shallow and deep subfaults might be more informative of different slip-weakening behaviors than the estimated values of  $d_c$ , which could be biased as previously indicated. Their results indicate that shallow subfaults are systematically characterized by a slow rate of weakening, which they attribute to constitutive properties that might stabilize the rupture.

The 0.8 m slip-weakening distance obtained by Olsen *et al.* (1997) for the 1992 Landers earthquake is not well constrained (as they themselves noted) because these authors did not explore the range of strength excess and slip-weakening distance consistent with Wald and Heaton's (1994) kinematic model. Olsen *et al.* (1997) found a dynamic model which duplicated Cotton and Campillo's (1995) rise times, but our models A and B have the same rise time and show that the rise time parameter is not adequate to discriminate between long and short slip-weakening distance models in the range of variation considered in our and their study. However, we should clarify that they formulated the problem in terms of constant constitutive parameters such as the peak and the frictional stress, and a nonuniform initial shear stress.

Consequently, with the possible exception of Ide and Takeo's (1997) 1-m slip-weakening distance (slow rate of weakening) for the shallow Nojima Fault, none of the presently available determinations of slip-weakening distance or strength excess based on waveform inversion is unique.

#### Implications for Resolution of Source-Time-Function Parameters

The lack of resolution of constitutive parameters such as strength excess and slip-weakening distance from narrowband waveform inversions implies that the inferred parameters of the source-time function, such as peak-slip velocity, duration, and time function, are also poorly constrained.

The peak-slip velocity is an important strong-motion parameter in broadband simulations of ground motion for engineering applications. Yet, from the comparison of the STF's of model A and model B (Fig. 9), it is evident that constraining the slip-rate to be temporally smooth would likely cause its peak value to be underestimated from narrowband ground-motion data.

The STF's of model A and model B are both consistent with a slip-weakening behavior, and therefore the slip duration at a point on the fault in both models is controlled by the rupture dimension. In the two models, the total slip at each subfault is about the same, but in model A large-slip

velocities occur over a much shorter period of time (controlled by  $T_c$ ) than in model B. This difference affects the resulting ground motions only at high frequency, implying that the rates of growth and decay of slip velocity are not uniquely constrained from narrowband ground-motion data (Spudich and Archuleta, 1987). Nonetheless, the shape of the STF may greatly affect the ground motion at high frequency.

As shown in Figures 9 and 10 and as expected from theory of crack propagation (e.g., Andrews, 1985), as the crack length increases, the STF's are characterized by a relatively high peak-slip velocity and fast decay, and by a tail of low-slip velocity. The total duration of slip is still controlled by the rupture dimension, but a very low-slip velocity tail might be seismically invisible, causing these STF's to be seen as narrower slip functions such as slip pulses. Therefore, in cases in which the recorded ground motions are dominated by energy radiated by regions of the fault far away from the hypocenter, an inferred slip-pulse behavior over the entire fault might be the result of a biased resolution. In these cases, a simple slip-weakening model characterized by a short slip-weakening distance might be consistent with observed slip pulses and the associated important directivity effects and their significant hazard.

For the 1984 Morgan Hill earthquake for which there is strong evidence of a short rise time (Beroza and Spudich, 1988), Beroza and Mikumo (1996) have shown that a dynamic model governed by a critical fracture criterion can explain the observed waveforms. The slip-rate function associated with a critical fracture criterion can be considered as an approximation of the slip-rate function resulting from a slip-weakening model with an extremely short  $d_c$ . Therefore, assuming that slip-weakening is the prevailing constitutive behavior during dynamic rupture, a short rise time and pulse-like slip function needed to fit the observed ground motion (Heaton, 1990) might suggest that  $d_c$  is short, of the order of cm or less, at least over the region of the fault that has the largest contribution to the recorded seismic radiation.

#### How Can We Reduce the Dynamic Nonuniqueness?

One way to learn more about slip-weakening distance would be to attempt to simulate the ground-motion spectra in the frequency band above that in which waveform modeling is possible. Figure 8 shows that ground-motion spectra are systematically elevated at high frequencies when slip-weakening distance is short. It might be possible to combine waveform and spectral criteria into the misfit function of future inversions.

Ohnaka and Yamashita (1989) found theoretical and experimental relations among strong-motion parameters and frictional properties of the fault. By analyzing the high-frequency radiation, these relations might help narrow the space of dynamic solutions consistent with low-frequency data. We write their equation (50) that specifies the relation between the local breakdown time  $T_c$  and the yield strength  $\sigma^y$ , the rupture velocity  $v$  and the slip-weakening distance  $d_c$ :

$$T_c \propto \frac{C(v)}{v} \frac{\mu}{\sigma^y} d_c \quad (3)$$

where  $C(v)$  is a function of rupture velocity, and  $\mu$  is the shear modulus. From this relation they found that  $T_c$  becomes shorter as both  $v$  and  $\sigma^y$  get higher and as  $d_c$  becomes shorter. From equation (3) the ratio

$$\frac{d_c}{T_c \sigma^y} \propto \frac{v}{\mu C(v)} \quad (4)$$

is expected to be equal for models A and B because the rupture velocity and the elastic properties of the medium are identical. We investigate the validity of this relation for our models by calculating the corresponding ratio in equation (4) at each subfault using, instead of the yield strength, the respective values of strength excess, which is more relevant to seismic rupture propagation because the process is primarily controlled by stress differences. We estimate  $T_c$  from the slip and stress histories of the two models (Fig. 9). In Figure 13 we plot the value of equation (4) calculated at each subfault for model A along the  $x$  axis and that for model B along the  $y$  axis. From this plot we can conclude that the relation in equation (4) holds for the two models analyzed in this study, and from this result, we can envision a feasible way to reduce the ambiguity in inferring a dynamic description of the source consistent with observations.

Ohnaka and Yamashita (1989), on the basis of empirical and theoretical results, determined  $f_{\max}^s = 1/T_c$  as the cutoff frequency of the power spectral density of slip acceleration, and they argue that  $f_{\max} \leq f_{\max}^s$ , where  $f_{\max}$  is the upper limit of frequency of spectral band of strong-motion accelerograms recorded in the near-source region from an earthquake source (e.g., Brune, 1970; Ida, 1973; Hanks, 1982, Papageorgiou and Aki, 1983). Therefore, from relation (4) we can find a lower bound on the ratio between the strength excess and slip-weakening distance

$$\frac{(\sigma^y - \sigma^o)}{d_c} \geq f_{\max} \mathcal{T}(v, \mu) \quad (5)$$

where  $\mathcal{T}(\theta, \mu)$  is a function of rupture velocity and the shear modulus. Once estimates of rupture velocity and slip distribution for a given earthquake source are found by waveform analysis, we can speculate that the nonuniqueness of the corresponding source-dynamic description can be reduced by imposing the lower bound from equation (5) on the ratio of strength excess and slip-weakening distance that characterizes the fault-constitutive behavior.

A second way to resolve the trade-off of slip-weakening distance with strength excess might be to follow a suggestion made by Scholz (1988) and Ide and Takeo (1997) that slip-weakening distance is long where there are no aftershocks. Such an interpretation might resolve some paradoxes. Ide and Takeo (1996) correlated high-strength excess in the

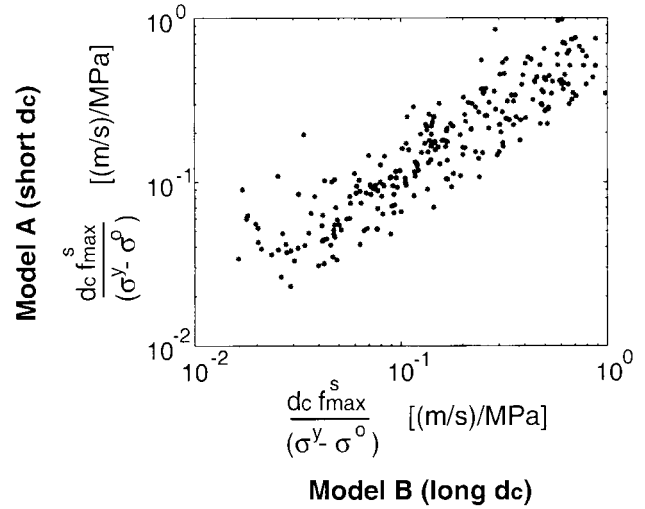


Figure 13.  $d_c f_{\max}^s / (\sigma^y - \sigma^o)$  for model A as a function of  $d_c f_{\max}^s / (\sigma^y - \sigma^o)$  for model B, where  $f_{\max}^s$  (Ohnaka and Yamashita, 1989). This figure shows that this ratio is roughly the same for both models. In fact, following Ohnaka and Yamashita (1989), this ratio is a function of the rupture velocity and shear modulus, and therefore is expected to be the same for both models. This constrain might be used to apply spectral analysis to broadband data to reduce the dynamic nonuniqueness discussed in the article.

1993 Kushiro-Oki earthquake with an abundance of aftershocks, whereas Fukuyama and Mikumo (1993) correlated high values of strength excess on the 1990 Izu-Oshima rupture surface with a lack of aftershocks. Note that, under the assumption of uniform initial-shear stress, the strength excess is an indicator of the relative distribution of peak stress over the fault plane only for the regions that slipped during the mainshock. For the areas with no slip, the strength excess is only a transient quantity, which might be simplistically considered as a lower bound of the local peak stress. The spatial correlation of Ide and Takeo (1996, their Plate 2) is rather problematic because they show a region of high-strength excess on the west part of the fault having many aftershocks, but they show regions of high-strength excess on the east corner of the fault having very few aftershocks and a region of low-strength excess around the hypocenter having very few aftershocks. Given this type of ambiguity in correlating strength excess and aftershock occurrence, the interpretation that regions of long slip-weakening distance rather than high-strength excess stop the rupture in regions where there are few aftershocks (Scholz, 1988; Ide and Takeo, 1997) might be preferable for both earthquakes. This possibility must not be ignored in future dynamic modeling of earthquake ground-motion data.

Moreover, we can obtain the distribution of the upper bound of slip-weakening distance over the fault plane from the distribution of slip implied by the smallest detectable aftershocks mapped over the fault plane where the mainshock occurs (Aki, 1987). Of course, such an estimate of the

upper bound should also account for the effects of the earthquake detection threshold, the time of surface contact on the slip-weakening distance (Nakatani, 1997; Aochi and Matsu'ra, 1999), and the structural changes of the fault surfaces occurring during rupture. Nevertheless, it would provide an estimate of the order of magnitude of the upper bound of slip-weakening distance that would certainly help reduce the nonuniqueness discussed in this article.

### Conclusions

At present, from strong ground motion studies, we have very little information about constitutive relations because, with a few possible exceptions, it is not clear that any waveform-modeling inversion of an earthquake source has resolved time scales shorter than the rise time.

The main result of our article is even more gloomy than the above conclusion, namely, waveform inversion of the best existing strong-motion-data set will probably be unable to determine the strength excess and slip-weakening distance uniquely. The fundamental ambiguity is that strength excess and slip-weakening distance have competing effects on rupture velocity, which largely controls the waveforms. In our inversion, we modeled the waveforms at periods that were one third of the rise time, and we found two rupture models, A and B, having different strength excesses and slip weakening distances, but both models produced waveforms that were essentially indistinguishable.

However, our work shows that fracture energy might be much more stably estimated from waveform inversions. Our models A and B had very similar fracture energies. Although we did not invert real data, our models A and B show that an event of the size of the Imperial Valley earthquake has a fracture energy of about  $2\text{--}6 \times 10^6 \text{ J/m}^2$ , which agrees well with the  $2 \times 10^6 \text{ J/m}^2$  obtained by Beroza and Spudich (1988) for the Morgan Hill earthquake. However, during the Imperial Valley event the rupture velocity was inferred to be supershear over a portion of the fault plane (Archuleta, 1984; Spudich and Cranswick, 1984), and in the models presented in this article, the rupture propagates at a subshear velocity. Due to this discrepancy, our estimate of apparent fracture energy probably overestimates that for the 1979 Imperial Valley earthquake. Previous theoretical studies explain why fracture energy might be a stable inversion parameter. These studies show that rupture velocity is controlled by fracture energy if stress drop is fixed. For a slip-weakening model, Ida (1972) and Andrews (1976a; 1976b) give analytical expressions that relate the apparent fracture energy to rupture velocity in the case of uniform stress drop. In these relations the rupture velocity is determined by the ratio of the apparent fracture energy  $G_c$  with a parameter that depends on the square of the stress drop. Thus, if we know the stress-drop distribution for an earthquake from its slip distribution obtained by a ground-motion inversion, the earthquake's rupture velocity will constrain the apparent fracture energy for the event. As a given kinematic model is

consistent with a broad range of values of strength excess and slip-weakening distance, so also is a given distribution of fracture energy nonuniquely related to these two parameters. Nevertheless, the fracture energy is a fundamental parameter for describing earthquakes in terms of the physics and provides important insights into the rupture process.

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## Appendix

### Method of Calculation

Our method uses a dynamic approach for the modeling and parameterization of the seismic source and is based on the boundary integral spontaneous-rupture code (Boatwright and Quin, 1986; Das and Kostrov, 1987; Quin and Das, 1989) that was incorporated into an inversion procedure by Spudich and Boatwright (1990) and Guatteri and Spudich (1996). First, we briefly describe the method to calculate spontaneous rupture. A more detailed description can be found in Guatteri and Spudich (1998). In this method a shear crack lies in the plane  $x_3 = 0$  in a Cartesian coordinate system. Let us discretize the crack plane onto a rectangular grid at locations  $ij$  (subfaults) and let the  $k$ -th time step be  $t_k$ . Let  $u_{aijk}$  be the  $\alpha$ -component of slip of the crack face at location  $ij$  and time step  $k$ , and let  $\tau_{aijk}$  be the  $\alpha$ -component of stress change at location  $ij$  and time step  $k$  caused by motions on the fault. The motions of the crack face at time step  $k$  is determined from stresses on the fault at previous time steps by:

$$u_{aijk} + \Delta t G_0 \tau_{aijk} = L_{aijk} \quad (\text{A1})$$

where  $\Delta t$  is the time step  $t_{k+1} - t_k$  and  $\Delta t G_0$  is the compliance of the medium (Andrews, 1985).  $L_{aijk}$  is the load (Andrews, 1985), and it is the dynamic quantity that includes the contribution to slip caused at time step  $k$  by stress changes at previous time step. Equation (A1) is the unloading curve, which relates stress change to total slip at time step  $k$ , depending on the load exerted on point  $ij$ . Given a particular value of load  $L_{aijk}$ , several pairs of stress change  $\tau_{aijk}$  and slip  $u_{aijk}$  can satisfy equation (A1); stress change and slip are uniquely specified only when a boundary condition is specified. In forward-dynamic modeling this bound-

ary condition is given by the assumed constitutive relation (or failure criterion) relating slip, slip-velocity, stress, and any other relevant physical quantities. Therefore, the slip and stress change that occur at a point on the fault at time step  $k$  are given by the intersection of the unloading curve and the curve that parameterizes the constitutive relation. The forward calculation proceeds by looping over all points on the fault for each time step, and for each fault point it determines the intersection of the unloading curve and the assumed friction law.

We convert the spontaneous rupture code into an inversion algorithm based on the idea that, rather than a completely specified constitutive law, we can impose the slip that satisfies other desired constraints as boundary condition to equation (A1), and solve this equation for the stress change consistent with the calculated value of load. In this way we find the slip and stress change that must occur at each time step in order to satisfy a given set of desired properties. At each point on the fault, the load  $L_{aijk}$  is determined from stress changes at previous time steps, according to equation (1) in Guatteri and Spudich (1998). For the first time step, the load is zero.

Let  $s$  be the subfault corresponding to the indices  $ij$ , and let us assume isotropic friction. This assumption means that the slip velocity is collinear with total traction (friction), so that the slip direction is predetermined at each time step. Therefore, we can remove the slip-direction parameter (the index  $\alpha$  in equation A1) from the problem, and for subfault  $s$  and time step  $k$  define the slip increment  $\Delta u_{sk}$  along the direction determined by the total traction vector. We use a linear programming algorithm to solve for the slip increments  $\Delta u_{sk}$  for the set of subfaults  $s$  that we place into a  $N_s$ -dimensional solution vector  $\mathbf{m}$  at time step  $k$ . At each time step we select constraints to apply to the motion of the fault on any point depending on the previous rupture history.

The general form of a linear minimization problem is stated as follows. Let  $\mathbf{x}$  be the solution vector composed by  $N$  independent variables; we find the  $\mathbf{x}$  that minimizes the objective function  $\psi = \mathbf{c}^T \mathbf{x}$  subject to the primary constraints

$$x_j \geq 0, \quad i = 1, \dots, N \quad (\text{A2})$$

and to equality constraints

$$\mathbf{B}\mathbf{x} = \mathbf{d}, \quad (\text{A3})$$

where  $\mathbf{d}$  is the  $N_d$ -dimensional data vector, and  $\mathbf{B}$  is the  $(N_d \times N)$  matrix that maps the model space ( $\mathbf{x}$ ) into the data vector  $\mathbf{d}$ . Inequality constraints can be easily included by means of slack variables (Luenberger, 1984). In the following we discuss the choice of the constraints and their implementation in a linear minimization problem.

The application of constraints allows us to obtain a set of dynamic models consistent with required properties. Our goal is to show that a broad range of dynamic models sat-

isfies important kinematic parameters such as rupture velocity, moment-rate function, and final slip. We accomplish this task by generating two source models satisfying the same kinematic constraints and subject to different dynamic properties.

It is important to note that at every time step  $k$ , the load vector  $L_{aijk}$  is completely determined by quantities calculated at previous time steps; this means that the information about the past rupture process constrains the solution at the current time step.

We separate the constraints into kinematic, smoothing, and dynamic constraints.

#### Kinematic Constraints

**K1. Total slip increment at time step  $k$  equals a given moment-rate function.** At time step  $k$  the moment rate function  $M_k$  of the reference model is applied as a constraint to

both models. We require that  $M_k = \sum_{s=1}^{N_s} \Delta U_{sk}$ .

**K2. Rupture velocity.** We impose the reference model's rupture-time distribution on both models by prohibiting slip on each subfault until a desired time.

#### Smoothing Constraints

**S1. Minimize the first difference between cumulative slip on adjacent subfaults.** At each time step  $k$  we smooth the cumulative slip on adjacent subfaults along the strike and dip of the fault.  $u_{s(k-1)}$  and  $u_{(s+1)(k-1)}$  are the cumulative slip at the previous time step at two adjacent subfaults, and  $\Delta u_{sk}$  and  $\Delta u_{(s+1)k}$  are the respective incremental slip (unknown) at the current time step. Let  $r_{s(s+1)k}$  be the first difference between cumulative slip on adjacent subfaults:

$$r_{s(s+1)k} = (u_{s(k-1)} + \Delta u_{sk}) - (u_{(s+1)(k-1)} + \Delta u_{(s+1)k}). \quad (A4)$$

The objective function  $\psi$  is the  $L_1$  norm of the vector of residuals  $\mathbf{r}$  formed by the  $r_{s(s+1)k}$  for all the pairs of adjacent subfaults that enter the solution vector  $\mathbf{m}$  at time step  $t_k$ . To express the  $L_1$  minimization problem in the standard form of the linear programming, we represent the residual vector  $\mathbf{r}$  as the difference of two vectors with nonnegative components (Das and Kostrov, 1990),  $\mathbf{r} = \mathbf{r}^+ - \mathbf{r}^-$ . Appending the vector  $\mathbf{r}$  to the vector of unknowns  $\mathbf{m}$ , we form the solution vector  $\mathbf{x} = (\mathbf{m} \mathbf{r}^+ \mathbf{r}^-)^T$ . This constraint together with the requirement K1 specify the basic formulation of our linear minimization problem. For each time step  $k$ , let  $h_q = u_{s(k-1)} - u_{(s+1)(k-1)}$ ,  $q = 1, \dots, N_q$  where  $N_q$  is the number of pairs of adjacent subfaults that enter the solution vector  $\mathbf{m}$ . Then, in equation (A3)  $\mathbf{d} = (M_k \mathbf{h})^T$  is the  $N_d = N_q + 1$  - dimensional data vector,  $\mathbf{x}$  is the  $N = N_s + 2 \times N_q$  - dimensional solution vector, and

$$\mathbf{B} = \begin{pmatrix} \mathbf{1}_{1 \times N_s} & \mathbf{0}_{1 \times N_q} & \mathbf{0}_{1 \times N_q} \\ \mathbf{D}_{N_q \times N_s} & \mathbf{I}_{N_q \times N_q} & -\mathbf{I}_{N_q \times N_q} \end{pmatrix}, \quad (A5)$$

where  $\mathbf{D}$  is the first difference matrix,  $\mathbf{I}$  is the identity matrix, and  $\mathbf{0}_{n \times m}$  and  $\mathbf{1}_{n \times m}$  are the  $n \times m$  matrices of zeros and ones.

From the following dynamic constraints, we calculate lower and upper bounds on the solution vector  $\mathbf{m}$ .

#### Dynamic Constraints

**D1. The stress drop is bounded.** At any time step  $k$ , stress drop can be calculated from the stress change calculated using equation (A1). We impose an upper bound on the stress drop in each subfault, which is the static stress drop of the reference model, so that the stress-drop distribution of the two extremal models has similar characteristics. Moreover, we can use this upper bound of stress drop to implement the constraint D3. The imposed maximum stress drop for the two extremal models follows the same spatial distribution, but is scaled differently in order to obtain the desired final-slip distribution.

**D2. Slip velocity is bounded.** We assume isotropic friction (i.e., slip velocity is always collinear with total traction). This is a fundamental physical constraint that applies at all points and all time steps and determines the slip direction at each time step based on previous conditions. From this requirement Spudich (1992, equation 4–7, 8b) derived an upper bound on the slip velocity.

**D3. The rate of slip-weakening is bounded.** We impose different bounds on the slope of the slip-weakening curve for the two extremal models. In particular, we force one model to have a steep slip-weakening curve (fast weakening) resulting in an average slip-weakening distance of  $\sim 0.3$  m (model A), and the second model to follow a slow weakening with an average slip-weakening distance of  $\sim 1$  m (model B). These values for slip-weakening distance are much larger than the estimates obtained from laboratory experiments (e.g., Dieterich, 1979; Okubo and Dieterich, 1989). However, in the context of waveform analysis and numerical-rupture simulation, in principle, only slip-weakening distances of the order of cm can be resolved.

For simplicity here we show how the bounds on the rate of weakening are imposed for a pure strike-slip case. The more general case in which the rake of slip is not fixed *a priori* is easily approached using the collinearity requirement between slip velocity and friction (constraint D2). At a given subfault  $s$  let  $\Delta \sigma_{\max}$  be the upper bound on stress drop from constraint D1,  $\sigma^y$  be the yield stress found as the value of stress at this subfault at the onset of slip, and  $d_{c\min}$  and  $d_{c\max}$  be the minimum and maximum slip-weakening distances, respectively, that we wish the resulting slip-weakening curve governing the dynamics of the model to have. From these parameters we can calculate the maximum rate  $\alpha_1$  and the minimum rate  $\alpha_2$  of weakening that we impose to our solution at each time step:

$$\begin{aligned} -\alpha_1 &= \frac{C(\sigma^y + \Delta \sigma_{\max})}{d_{c\min}} \\ -\alpha_2 &= \frac{C(\sigma^y + \Delta \sigma_{\max})}{d_{c\max}}, \text{ where } C = G_0 \Delta t. \end{aligned} \quad (A6)$$

From these we can derive an upper and lower bound to the incremental slip  $\Delta u_k$  that is the solution of our linear programming problem at time step  $t_k$  at the given subfault. In fact, once the subfault starts slipping, we want that

$$-\alpha_2 \leq \frac{T_k - T_{k-1}}{\Delta u_k} \leq -\alpha_1, \text{ where } T_k = C\tau_k, \quad (\text{A7})$$

and from equation (A1) substituting  $T_k = L_k - u_k$ , and  $u_k = u_{k-1} + \Delta u_k$  we obtain

$$\frac{T_{k-1} + u_{k-1} - L_k}{1 - \alpha_2} \leq \Delta u_k \leq \frac{T_{k-1} + u_{k-1} - L_k}{1 - \alpha_1}, \quad (\text{A8})$$

where we have omitted the  $s$  subscript for simplicity.

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