

Earthquake cycle, fault zones, and seismicity patterns in a rheologically layered lithosphere

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Abstract. We study the coupled evolution of earthquakes and faults in a model consisting of a seismogenic upper crust governed by damage rheology over a viscoelastic substrate. The damage rheology has two types of functional coefficients: (1) a "generalized internal friction" separating states associated with material degradation and healing and (2) damage rate coefficients for positive (degradation) and negative (healing) changes. The evolving damage modifies the effective elastic properties of material in the upper crust as a function of the ongoing deformation. This simulates the creation and healing of fault systems in the upper seismogenic zone. In addition to the vertically averaged thin sheet approximation we introduce a Green function for three-dimensional elastic half-space for the instantaneous component of deformation. The formulation accounts in an internally consistent manner for evolving deformation fields, evolving fault structures, aseismic energy release, and spatiotemporal seismicity patterns. These developments allow us to simulate long histories of crustal deformation and to study the simultaneous evolution of regional earthquakes and faults for various model realizations. To focus on basic features of a large strike-slip fault system, we first consider a simplified geometry of the seismogenic crust by prescribing initial conditions consisting of a narrow damage zone in an otherwise damage-free plate. For this configuration, the model generates an earthquake cycle with distinct interseismic, preseismic, coseismic, and postseismic periods. Model evolution during each period is controlled by a subset of physical properties, which may be constrained by geophysical, geodetic, rock mechanics, and seismological data. In the more generic case with a random initial damage distribution, the model generates large crustal faults and subsidiary branches with complex geometries. The simulated statistics depend on the space-time window of the observational domain. The results indicate that long healing timescale, τ_h , describing systems with relatively long memory, leads to the development of geometrically regular fault systems and the characteristic frequency-size earthquake distribution. Conversely, short τ_h (relatively short memory) leads to the development of a network of disordered fault systems and the Gutenberg-Richter earthquake statistics. For intermediate values of τ_h the results exhibit alternating overall switching of response from periods of intense seismic activity and the characteristic earthquake distribution to periods of low seismic activity and Gutenberg-Richter statistics.

1. Introduction

A decade-long surge of activity in the seismological and physics communities is aimed at resolving fundamental questions related to spatiotemporal patterns of earthquakes and faults. These studies typically employ numerical simulations of models based on different conceptual

frameworks. Recent summaries and classifications are given by, e.g., *Gabrielov and Newman* [1994] and *Ben-Zion et al.* [1999a]. Most models to date were confined to studies of seismic activity along a single or a few fault systems [e.g., *Robinson and Benites*, 1995]. A few works perform two-dimensional (2-D) calculations of elastostatic deformations in a thin plate to simulate respectively long histories of regional seismicity and the development of regional faults [*Sornette et al.*, 1994; *Ward*, 1996; *Cowie*, 1998, and references therein]. Such 2-D models, however, ignore the coupling of the seismogenic zone to other parts of

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the lithosphere and fault interactions involving the third dimension. Depth plays a major mechanical role through coupling of the upper seismogenic layer to a viscoelastic substrate. This may produce time-dependent inelastic deformation zones that spread in the ductile lower crust and load faults in the brittle upper layer [e.g., *Lehner et al.*, 1981; *Thatcher*, 1983; *Li and Rice*, 1987; *Ben-Zion et al.*, 1993; *Reches et al.*, 1994]. A second important shortcoming of the foregoing models is that they employ either fixed imposed faults [*Ward*, 1996] or material properties that are constant in time [*Sornette et al.*, 1994], and they thus neglect the fact that the geometry and rheological properties of fault systems evolve with the ongoing deformation [e.g., *King*, 1983; *Andrews*, 1989; *Scholz et al.*, 1993]. *Heimpel and Olson* [1996] simulated lithospheric rifting in models with time-independent properties and approximated Elsasser-type interactions (see section 2) with the depth dimension. Since stress transfer mechanisms in a 3-D rheologically layered solid and the evolution of fault properties with deformation can be important over timescales longer than a few great earthquake cycles, the existing models do not provide conceptually complete frameworks for studying long histories of crustal activity.

In the present work we attempt to overcome the above shortcomings by using a model consisting of an elastic upper layer governed by damage rheology [*Lyakhovsky et al.*, 1997 a, b] over a Maxwell viscoelastic substrate. The calculations employ vertically averaged variables of the thin sheet approximation for the viscous component of motion and a 3-D elastic Green function for the elastic response of the model to deformation. The damage rheology accounts for the creation, evolution, and possible healing of fault zones in the upper crust. The damage approach assumes that the density of cracks is uniform over a length scale much larger than the crack length, yet much smaller than the scale of the macroscopic problem considered. In a specimen under rock mechanics laboratory conditions, microcrack density can be measured over a subcentimeter scale and considered as distributed damage [*Moore and Lockner*, 1995]. On a plate boundary length scale, individual faults that extend to a kilometer length may be considered as distributed damage. The distributed cracks are much smaller than the size of a representative volume, in which crack density is considered homogenous. Hence we define an intensive damage variable α representing the ratio between the elastic shear modulus of a spatial domain relative to the modulus of an ideal crack-free solid. Two aspects of the physics of damage are treated by the present model: (1) a mechanical aspect, the sensitivity of the macroscopic elastic moduli to distributed cracks and to the type of loading, and (2) a kinetic aspect, damage evolution (degradation/recovery of elasticity) in response to ongoing deformation. The theory of the damage rheology and constraints to damage coefficients from laboratory data are discussed in detail by *Lyakhovsky et al.* [1997 a, b] and A. Agnon et al. (manuscript in preparation) and are briefly presented in Appendix A.

In the following, we present theoretical developments relevant to the large-scale structure of the model (layered elastic/viscoelastic half-space). The developed framework allows us to simulate long histories of crustal deformation and to study the simultaneous self-organization of regional earthquake and faults. The model formulation is used to calculate various examples of geodetic fields, evolving upper

crust properties, and seismicity patterns and to perform a basic parameter-space study of different dynamic regimes.

2. Model of Crustal Stress Distribution

Strain accumulation and release at a strike-slip plate boundary has been discussed by various authors [e.g., *Sibson*, 1982; *Meisner and Strehlau*, 1982; *Li and Rice*, 1987; *Ben-Zion et al.*, 1993; *Reches et al.*, 1994]. The shallow portion of the lithosphere is generally characterized as elastic and brittle, while the deeper material is assumed to undergo plastic shear flow and creep due to high temperature and pressure. In this work the seismogenic upper crust is governed by damage rheology and is coupled viscoelastically to the substrate, where steady mantle flow drives the deformation. We use different prescribed loads representing various types of mantle flow, ranging from localized step function underneath a single major fault to distributed loading for simulations of regional evolutions of earthquakes and faults.

Most studies of surface deformation around a preexisting plate boundary [*Li and Rice*, 1987; *Ben-Zion et al.*, 1993; *Reches et al.*, 1994] approximate the velocity profile in the mantle with a step-like function. In these models the mantle on either side of the boundary moves with fixed, opposite polarity, velocity parallel to the major fault trace. Such a condition assumes that the mantle flow follows the fault trace, although localization in the asthenosphere is not an anticipated feature. Typical simulations of mantle convection do not lead to strong horizontal shear flow localization even in the colder lithosphere.

Elsasser [1969] developed a vertically averaged thin sheet approximation for a brittle upper crust over a viscous substrate. *Rice* [1980], *Lehner et al.* [1981], and *Li and Rice* [1987] provided a generalized Elsasser model replacing the viscous rheology of the substrate with viscoelasticity. A few attempts have been made to calculate crustal deformation in a 3-D model with a depth-dependent rheology [e.g., *Ben-Zion et al.*, 1993; *Reches et al.*, 1994]. These simulations used the finite element code ABAQUS, and they are very demanding computationally even for modern supercomputers. The results from those studies show that outside the space-time vicinity of a large earthquake, the 3-D calculations do not differ much from those obtained by the Elsasser and

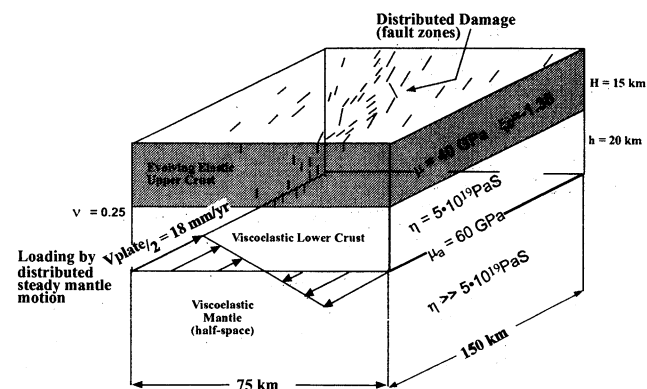


Figure 1. A layered elastic/viscoelastic half-space model. The brittle upper crust ($H=15$ km) is governed by damage rheology model and is coupled to a viscoelastic lower crust ($h=20$ km) driven by mantle movement (V_{plate}).

generalized Elsasser models. *Reches et al.* [1994] presented a direct comparison between a thickness-averaged fault-parallel velocity of the elastic layer from 3-D calculations and the analytical solution of *Li and Rice* [1987] for the generalized Elsasser model. After ~50 years into the earthquake cycle the difference between the 2-D and 3-D models is negligible, and at such times both models fit the available geodetic data with about the same accuracy. However, the calculations of *Reches et al.* [1994] and previous results of *Rice* and coworkers summarized by *Ben-Zion et al.* [1993] show that the generalized Elsasser model is not a satisfactory approximation to deformation in the early part of the earthquake cycle. In section 2.1 we discuss the main features of the generalized Elsasser model and present a hybrid model incorporating 3-D elastic Green function. The hybrid model provides a good approximation of the deformation field even in spatiotemporal domains close to the occurrence of large earthquakes, while being computationally much more efficient than a fully 3-D model.

2.1. Generalized Elsasser Model

Figure 1 shows a model consisting of an elastic upper crust governed by damage rheology over a viscoelastic lower crust with imposed basal loading. For a damage-free upper crust the model corresponds to the framework of *Rice* [1980], *Lehner et al.* [1981], and *Li and Rice* [1987]. The horizontal components of a stress tensor σ_{km} averaged over a thickness H of the upper crust are

$$\sigma_{km}(x, y) = \frac{1}{H} \int_{-H}^0 \sigma_{km}(x, y, z) dz \quad (1)$$

Substituting (1) into the 3-D equation of equilibrium gives for the crustal stress distribution,

$$\frac{\partial \sigma_{km}}{\partial x_m} = \frac{\tau_k}{H} \quad (2)$$

where τ_k is shear traction acting at the boundary between the upper and lower crust layers. This shear traction satisfies an equation of motion of the lower crust, which for a Maxwell viscoelastic element is [*Li and Rice*, 1987]

$$\frac{b}{\mu_a} \frac{\partial \tau_k}{\partial t} + \frac{h}{\eta} \tau_k = \frac{\partial u_k}{\partial t} - V_{plate}^{(k)} \quad (3)$$

where h , η , and μ_a are thickness, viscosity, and rigidity of the lower crust layer, respectively; $V_{plate}^{(k)}$ is the k -th component of the steady mantle velocity, and b is a scalar discussed below. Expression (3) includes two different terms on the left-hand side. The first describes approximately an instantaneous elastic response of the lower crust to evolving displacement ($\tau_k = \mu_a u_k / b$), and the second corresponds to viscous response of a layer with different velocities on its upper and lower boundaries ($\tau_k = \eta / h (\partial u_k / \partial t - V_{plate}^{(k)})$). A term $\rho g H \nabla S$, with ∇S giving the slope of the lower-upper crust interface, should be added to the viscous response if the upper crust has a variable thickness. Such cases are not considered here. The coupled equation of stress distribution, obtained by putting (2) into (3), is

$$\frac{Hb}{\mu_a} \frac{\partial}{\partial t} \frac{\partial \sigma_{km}}{\partial x_m} + \frac{Hh}{\eta} \frac{\partial \sigma_{km}}{\partial x_m} = \frac{\partial u_k}{\partial t} - V_{plate}^{(k)} \quad (4)$$

Lehner et al. [1981] showed that a vertically averaged stress drop in a homogeneous upper crust governed by (4) gives a

displacement similar to a mode III crack in an elastic half space if $b \approx (\pi/4)^2 H$. However, this is associated with an exponential decay of the elastic displacements with distance from the fault, while the exact elastic solution follows an algebraic decay. For long-term deformation the elastic field is only a small perturbation around viscous deformation, and the difference between exponential and algebraic decay of the elastic component is not very important. A comparison of deformations generated by an imposed large model earthquake in the generalized Elsasser framework [*Li and Rice*, 1987] and full 3-D simulations [*Reches et al.*, 1994] shows that the former overestimates displacement components have relatively short horizontal wavelengths. However, the difference in thickness-average velocity profile at times larger than tens of years after the earthquake is negligible.

To understand the spatiotemporal evolution of deformation associated with the generalized Elsasser model, we analyze the relaxation time of structures with different length scales by using a Fourier decomposition of a 1-D version of (4). We simplify (4) by assuming that the upper crust obeys Hooke's elasticity and that the deformation field (including imposed plate velocity) has only a "y" component of motion, which depends only on the "x" coordinate (antiplane strain). All material parameters are assumed constant. For these conditions, equation (4) reduces to

$$\frac{Hb\mu}{\mu_a} \frac{\partial}{\partial t} \frac{\partial^2 u}{\partial x^2} + \frac{Hh\mu}{\eta} \frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial t} - V_{plate}^{(k)} \quad (5)$$

For a zero plate motion ($V_{plate}=0$) we look for a basic solution in the form $u(x, t) = f(t) \varphi(x)$. With this equation (5) becomes

$$\frac{Hb\mu}{\mu_a} \dot{f} \varphi'' + \frac{Hh\mu}{\eta} f \varphi'' = \dot{f} \varphi \quad (6)$$

which may be separated into two equations:

$$\frac{Hb\mu}{\mu_a} \dot{f} + \frac{Hh\mu}{\eta} f = \frac{\varphi}{\varphi''} = -\lambda^2 \quad (7)$$

This procedure gives a basic solution to (5) in the form

$$\varphi = A \cdot \exp\left(i \frac{x}{\lambda}\right) \quad (8)$$

$$\dot{f} = B \cdot \exp\left(-\frac{hH\mu/\eta}{\lambda^2 + bH\mu/\mu_a} t\right)$$

According to this solution, the relaxation time of spatial structure with a horizontal wavelength λ is

$$t^* = \frac{\lambda^2 + bH\mu/\mu_a}{hH\mu/\eta} \quad (9)$$

In the short wavelength limit ($\lambda \rightarrow 0$) the relaxation time approaches $b\eta/h\mu_a$. For long wavelengths ($\lambda \rightarrow \infty$) we obtain $t^* = \lambda^2 \eta / hH\mu$, which also represents the relaxation time for the original Elsasser model corresponding to $b=0$. Figure 2 shows how the relaxation time changes as a function of wavelength between these limits. Figure 2 also shows the relaxation time for a new hybrid model described in section 2.2 and used later in our work.

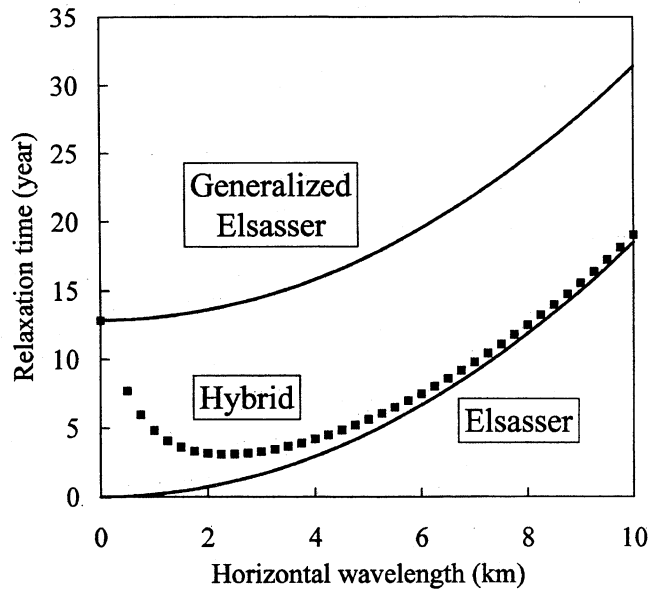


Figure 2. Relaxation time versus horizontal wavelength for the Elsasser and generalized Elsasser models (heavy lines). The numerically calculated relaxation time for the hybrid model (symbols) is similar to that of the generalized Elsasser model for very short length scales, but it approaches the Elsasser solution for long wavelengths (viscosity of the lower crust $\eta = 5 \cdot 10^{19}$ Pa s).

2.2. A Hybrid Model Incorporating 3-D Green Function

As mentioned above, the substitution $\tau_k = \mu_a u_k / b$ essential to the generalized Elsasser model does not reproduce correctly the elastic component of deformation of the lower crust. Our proposed modification of that model follows the usual seismological assumption [e.g., Kasahara, 1981; Stein et al., 1994] that the instantaneous response of the Earth to a sudden stress redistribution is accommodated by an elastic half-space. This assumption is realistic, since the relaxation of strain is not instantaneous even with a lower crust of extremely low viscosity. For example, with a viscosity of 10^{19} Pa s the Maxwell relaxation time is of the order of tens years.

We represent the horizontal displacement field at the boundary between the upper and lower crust layers as a sum of two components: elastic ($u_k^{(e)}$) due to instantaneous deformation in the upper crust and viscous ($u_k^{(v)}$) due to slow flow in the lower crust:

$$u_k = u_k^{(v)} + u_k^{(e)} \quad (10)$$

Equating the stress on the both sides of the lower-upper crust interface, the rate of viscous flow is

$$\frac{\partial u_k^{(v)}}{\partial t} = \frac{h}{\eta} \tau_k + V_{plate}^k \quad (11)$$

For the elastic part of the displacement field we use a 3-D Green function (G_{kn}) in an elastic half-space. The elastic displacement at the boundary surface (S) between the lower and upper crust layers is calculated as a convolution of G_{kn} with forces acting at S :

$$u_k^{(e)} = \iint_S G_{kn} \tau_k ds \quad (12)$$

Taking partial derivative of (12) with respect to time,

combining the result with (10) and (11), and substituting back into (2) gives the equation of motion for the hybrid model:

$$H \iint_S G_{kn} \frac{\partial}{\partial t} \frac{\partial \sigma_{nm}}{\partial x_m} ds + \frac{hH}{\eta} \frac{\partial \sigma_{nm}}{\partial x_m} = \frac{\partial u_k}{\partial t} - V_{plate}^{(k)} \quad (13)$$

A strictly rigorous Green function for our calculations would account for the varying topography and varying elastic properties simulated by our model (see section 3) and for the fact that the forces operate at a depth H ($=15$ km here) below the free surface. However, since elsewhere we use in the model the thin sheet approximation (i.e., vertically averaged variables), we employ a Green function for a homogeneous elastic half space [Landau and Lifshitz, 1970] multiplied by the thickness of the upper layer, instead of precise integration along vertical direction inside the upper crust. The Green function components become

$$\begin{aligned} G_{xx} &= \frac{H}{2\pi\mu_a} \left(\frac{1-\nu}{r} + \frac{\nu x^2}{r^3} \right) \\ G_{yy} &= \frac{H}{2\pi\mu_a} \left(\frac{1-\nu}{r} + \frac{\nu y^2}{r^3} \right) \\ G_{xy} &= \frac{H}{2\pi\mu_a} \frac{\nu xy}{r^3} \end{aligned} \quad (14)$$

where r is a radius vector from the origin ($r^2 = x^2 + y^2$) and ν is the Poisson ratio.

Equation (13) differs from (4) of the generalized Elsasser model by incorporating a 3-D elastic response to rapid stress variations. Below we refer to the framework described by (13) as a "hybrid model". Similar to the analysis in the context of (4-9), we may calculate the relaxation time of spatial structures in the hybrid model with horizontal wavelength λ from (13) as

$$t^* = \frac{\lambda^2 + H\mu \tilde{G}_{xx}(\lambda)}{hH\mu/\eta}$$

where $\tilde{G}_{xx}(\lambda)$ is the Fourier transformation of the Green function. The solid squares in Figure 2 show the calculated relaxation time for different wavelengths in the hybrid model incorporating the Green function (equation (14)). The relaxation time of this model is similar to that of the generalized Elsasser model for very short length scales, but it approaches the basic Elsasser solution for long wavelengths. The spectral response of the hybrid model, bridging the Elsasser and generalized Elsasser models, corrects the shortcoming of the latter framework associated with overestimation of the lifetime of localized stress anomalies after brittle failures. This shortcoming leads to significant local deviations from 3-D calculations [e.g., Ben-Zion et al., 1993; Reches et al., 1994] during the first few years after rapid slip events. This is illustrated in Figure 3 where we compare fault parallel velocity at two normal distances from the fault as a function of time, calculated with the generalized Elsasser model [Li and Rice, 1987, equation A6] and our hybrid model.

3. Upper Crust Deformation and Damage Evolution

Our model of a lithosphere, incorporating upper crust material governed by damage rheology over a viscoelastic

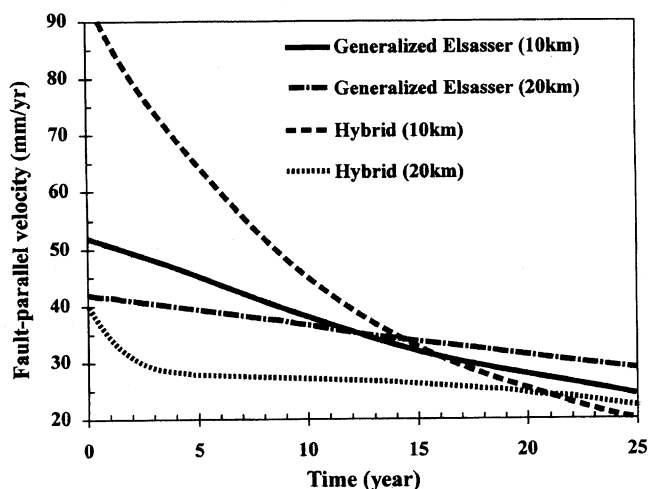


Figure 3. Fault-parallel velocity at two normal distances 10 and 20 km from the fault as a function of time after a seismic event, calculated with the generalized Elsasser model [Li and Rice, 1987, equation (A6)] and simulated with the hybrid model.

substrate, can simulate four stages of a seismic cycle: preseismic, coseismic, postseismic, and interseismic deformations. These stages may be anticipated from the behavior of a viscoelastic damaged material: a stick-slip behavior appears if the viscous relaxation time of the nondamaged medium τ_r is longer than the loading time scale τ_L (inverse strain rate) [Lyakhovsky *et al.*, 1997a]. The slip instability corresponds to the coseismic stage. The interseismic stage, spanning most of the stick period between the postseismic and preseismic stages, is represented by a fast healing and localization of damage, respectively (A. Agnon *et al.*, manuscript in preparation).

The hybrid version of the lithospheric model (Figure 1) generates an earthquake cycle under constant loading. Each element of a two-dimensional horizontal mesh may follow a stick-slip cycle, and the collective behavior of a group of elements model the faulting process and macroscopic earthquake failure. When deformation in an element reaches a threshold state, damage starts to increase with degradation of elastic moduli and concentration of elastic strain. This spatially localized initial perturbation in strain corresponds to the preseismic stage, with small geodetic signals at the free surface (possibly below the typical detection limit). A critical damage level for brittle instability marks the onset of the coseismic stage during which the local stress drops. This local stress drop may provide a nucleus for rupture that propagates by reloading the other elements to the critical level. The rupture propagation process continues as long as one or more elements experience brittle failure. When no element sustains brittle instability, the model earthquake ends. The brittle failure leads to an increase in the cumulative irreversible plastic strain, corresponding to the coseismic slip in models that idealize the fault zone as a surface.

The instability during a stress drop requires that we specify the subsequent stress conditions. Earthquake slip histories inferred from observed seismograms [Heaton, 1990], laboratory experiments [e.g., Brune *et al.*, 1993; Anoushehpour and Brune, 1999], theoretical models [e.g., Mora and Place, 1994; Andrews and Ben-Zion, 1997], and a variety of geophysical observations summarized by Ben-Zion

and Andrews [1998] suggest that earthquake ruptures propagate in the form of narrow slip pulses associated with strong local dynamic stress drops. A complete local stress drop is a physical consequence of the damage mechanics approach since the instability is associated with a vanishing elastic modulus [Lyakhovsky *et al.*, 1997a]. Averaged stress drop along a sliding narrow damage zone reproduces rate- and state-dependent friction (A. Agnon *et al.*, manuscript in preparation). Thus, following a model earthquake, the deviatoric stress is fully dropped locally and the crustal stress is rendered lithostatic, favorable for healing of damage in the postseismic stage. This stage may last from a fraction of a year to a few years, depending on the rate of material recovery.

The imposed plate velocity distribution, the length scales for the lithospheric layers, and the rheological coefficients of each layer comprise the parameters of the model. Some of the parameters can be constrained directly from geophysical data. The structural and kinematic values chosen follow a simplified Californian crust with a representative thickness of 35 km [Fuis and Mooney, 1990] sheared between plates moving at the surface with differential velocity of 36 mm/yr. Seismic activity is mostly restricted to the upper 10–15 km. We therefore use $H=15$ km and $h=20$ km in the simulation (Figure 1). From seismic refraction and other data [Fuis and Mooney, 1990; Mooney *et al.*, 1998] the average rigidity of the crust is estimated at 35–40 GPa for the upper crust and 40–70 GPa for the lower crust and upper mantle. Since the elastic moduli weaken with damage, a reference upper bound of $\mu=40$ GPa is chosen for damage-free upper crust rigidity. Below we constrain simulation results with geodetic data to obtain lower crust viscosity (η) and rigidity (μ_d). We also provide additional details on each stage of deformation evolution and discuss the role of the remaining parameters. We begin the survey of the parameter space with a simplified model that contains a single straight preexisting fault zone.

3.1. A Single Fault System

To study the model setting in incremental complexity level, we start with a step-like profile of the mantle velocity driving the fault. This profile leads to strain localization and damage evolution in a narrow zone above the step that is set in the center of the model ($x=0$), while other regions in the crustal model develop no damage. With an appropriate choice of boundary conditions the numerical simulation is required only in a small portion of the space around the fault zone. The interaction of the model region with the outer part of the lithosphere is represented by the boundary conditions at the horizontal edges of the model. Periodic boundary conditions at the edges normal to the fault trace represent infinite repetitions of the fault segment along strike. Conditions at the edges parallel to the fault zone should mimic steady plate motion away from the fault zone. Andrews [1978] and Wdowinski and O'Connell [1990] give useful discussions of the interaction between fault slip and the chosen boundary conditions. The condition ($V=V_{\text{plate}}$ for $x \rightarrow \infty$) may be replaced for a finite model as either (1) constant and uniform fault-parallel velocity ($V=V_{\text{plate}}$ for $x=\pm L$) or (2) constant and uniform shear stress ($\sigma_{xy}=\text{const}$ for $x=\pm L$). These seemingly equivalent boundary conditions generate very different solutions for fault-parallel velocity and stress distributions during the interseismic period. To illustrate the different behavior under the two types of boundary conditions, we

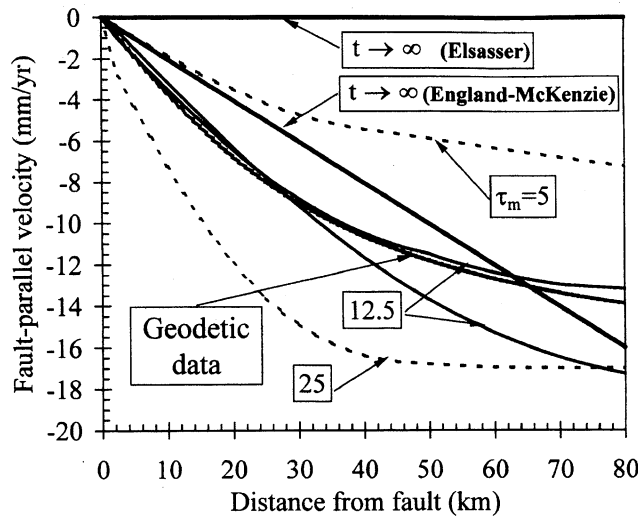


Figure 4. Velocity profiles 100 years after a large earthquake simulated for model relaxation times $\tau_m=5$, 12.5, and 25 years. The shaded line represents the best fit of geodetic data with dislocation model after *Savage* [1990]. Two asymptotic solutions ($t \rightarrow \infty$) for Elsasser and England-McKenzie boundary conditions are also shown.

examine the quasi-static limit appropriate for the long-term ($t \rightarrow \infty$) asymptotic solution by neglecting the temporal derivative on the left-hand side of (13). In the case of constant velocity boundary condition the velocity profile in the model is a straight line ($V(x)=V_{\text{plate}} x/L$) between zero velocity at the locked fault zone ($V=0, x=0$) and plate velocity at the boundary ($V=V_{\text{plate}}, x=\pm L$). This leads to unbounded increase of stress in the upper crust independently of the viscosity of the lower crust and the rate of motion in the upper mantle. *England and McKenzie* [1982], who used a thin sheet viscous model for the problem of long-term continental deformation assumed a similar decoupling between the plate and the substrate. This “England-McKenzie loading” does not provide an appropriate description for relatively short-term brittle-elastic behavior of the upper crust. By contrast, the constant stress boundary condition leads to zero velocity profile and uniform stresses equal the sum of the stress applied at the boundary and the loading from the mantle. In this case the stress in the model region depends on the viscosity of the lower crust and the rate of motion at the bottom of the model. This “Elsasser loading” [*Elsasser*, 1969] corresponds to a situation where the mantle-crust interaction is an essential part of the crustal processes. Since our simulations include short-term processes of brittle failure in the upper crust, we use the Elsasser loading.

During the interseismic period the damage is approximately frozen, and damage evolution does not significantly affect the strain rate of the upper crust. Thus the velocity profile in this period is governed mainly by two model parameters: the imposed mantle velocity and the viscosity of the lower crust. In the following, the mantle velocity is chosen to give far field surface velocity of $1/2 V_{\text{plate}} = 18$ mm/yr, hence long-term slip velocity is 36 mm/yr. Figure 4 shows simulated surface velocity profiles during the interseismic stage, 100 years after the last seismic event, for various relaxation times (τ_m) defined as

$$\tau_m = \frac{\pi^2}{16} \frac{H}{h} \frac{\eta}{\mu_a}$$

The definition of τ_m is based on the solution of the generalized Elsasser model for the elastic layer (equation (9) with $\lambda=0$). The relaxation times used in Figure 4 are $\tau_m=5$, 12.5, and 25 years. The lower value corresponds to $\eta=2.5 \times 10^{19}$ Pa s and $\mu_a=60$ GPa. The intermediate one corresponds to $\eta=5 \times 10^{19}$ Pa s and $\mu_a=60$ GPa, or $\eta=3 \times 10^{19}$ Pa s and $\mu_a=40$ GPa. The value $\tau_m=25$ years corresponds to $\eta=10^{20}$ Pa s and $\mu_a=60$ GPa.

Observed interseismic fault parallel velocities in geodetic measurements are particularly easy to model in the case where the fault is long and straight and the deformation is uniform in the direction of fault strike. In that case a screw dislocation in an elastic half-space is sufficient to model the observed velocity field at the free surface [*Savage*, 1980] as

$$V = V_0 \frac{1}{\pi} \arctan\left(\frac{x}{D}\right)$$

This profile, shown in Figure 4 with a shaded line for slip velocity $V_0=36$ mm/yr and dislocation depth $D=30$ km, provides the best fit [*Savage*, 1990] to interseismic geodetic data for the San Andreas Fault in the Transverse Ranges of southern California. *Lisowski et al.* [1991] show that a model with one dislocation fits the geodetic data even better than a multidislocation model. They also show that this fit corresponds to the middle part of the earthquake cycle time T ($0.4-0.6 T$) by comparison with the compiled data of *Thatcher* [1983].

Our model does not produce an exact arctangent velocity profile. On the basis of the solution of *Chinnery* [1961] the fault parallel elastic component of the displacement (12) with the Green function (14) generated by a stress drop localized along a straight fault follows an arctangent profile [*Kasahara*, 1981, equation 4.30]. The deformation field in our model has additional nonelastic components. Nevertheless, the calculated velocity profiles in Figure 4 at 100 years after the earthquake (the middle part of simulated 200-year earthquake cycle) are very similar to arctangent curves. As shown in Figure 1, we consider a model with $H=15$ km, $h=20$ km, and a horizontal extent of $150 \text{ km} \times 75 \text{ km}$. The shear modulus and the Poisson ratio of the damage-free upper crust material are $\mu=40$ GPa and $\nu=0.25$, respectively, and the Poisson ratio of the lower crust is $\nu=0.25$. We try two different values of the lower crust shear modulus $\mu_a=40$ and 60 GPa, and three different values of the lower crust viscosity $\eta=10^{20}$, 5×10^{19} , and 2.5×10^{19} Pa s. The line with $\tau_m=12.5$ years ($\eta=5 \times 10^{19}$ Pa s, and $\mu_a=60$ GPa) fits well the geodetic arctangent model. We use these parameters for all simulations to follow.

The interseismic period lasts until the current strain diagonality ξ (see Appendix A) is below a critical value ξ_0 separating states of damage increase (degradation) and decrease (healing) [*Lyakhovsky et al.*, 1997a]. This coefficient was estimated to vary between -0.8 and -1.0 using laboratory data on damage onset, Coulomb friction, and internal friction. In the current simulation, ξ_0 together with a prescribed width of the damage zone control the duration of the interseismic period and therefore the length of the earthquake cycle for a model with a single fault. In this work we use triangular elements of $2.5 \text{ km} \times 2.5 \text{ km}$. This gives a lower bound to the fault zone width in the simulations. Computational limitations prevent us from using a smaller

grid size that maybe more appropriate for natural fault zones. A smaller grid size would define the lower magnitude cutoff for the simulated earthquakes, mildly affect our choice of ξ_0 , but not the main conclusions of this work. The grid size dependency of some results is discussed further below.

Exploratory simulations indicate that for the prescribed 36 mm/yr rate of plate motion and average (effective) lithostatic (confining) pressure $p=(\rho-\rho_w)gH/2=0.1$ GPa (using $\rho=2500$ kg/m³ and $\rho_w=1000$ kg/m³), the shear strain in the upper crust never reaches values of ξ_0 inferred from the laboratory data for typical intact crustal rocks. Figure 5 shows that the change of current strain diagonality ξ with time during the inter-seismic period is very small. After ~ 100 years, ξ is almost constant, and thus a small variation in the material property ξ_0 has strong effects. With the increase of ξ_0 from -1.40 to -1.35 the length of the cycle increases by more than twice, from ~ 80 years to ~ 190 . Further increase of ξ_0 gives even a stronger incremental effect. As shown in Figure 3 of Lyakhovsky *et al.* [1997a], $\xi_0 = -1.35$ corresponds to an internal friction of ~ 0.3 for the Poisson ratio equal to 0.25. It is reasonable to expect that the value of internal friction characterizing a wide insitu gouge zone with internal cracks (i.e., damage) is smaller than laboratory values associated with small rock samples. A low value of friction coefficient is compatible with the lack of localized frictional heat along the San Andreas Fault [e.g., Brune *et al.*, 1969, Lachenbruch and Sass, 1973], calculations of changes in seismicity rates [Simpson and Reasenber, 1994] and analysis of aftershock mechanisms [Zoback and Beroza, 1993] following the Loma Prieta earthquake, simulated b-values on faults in elastic half-space [Robinson and Benites, 1995], and calculations of stress transfer favoring observed earthquake histories [e.g., Harris and Simpson, 1996; Stein *et al.*, 1997].

The damage model of brittle material does not include any prescribed surfaces that represent planar faults. Instead, we have fault zones associated with regions of high damage having a minimum width equal to the size of one numerical element. While a finite fault zone width is more realistic than a fault surface idealization, the existence of a prescribed minimum fault zone width makes some simulation results, like details of the evolving fault zone geometry and minimum model earthquake, dependent on the numerical cell size. This also renders our model inherently discrete in the sense of Rice [1993] and Ben-Zion and Rice [1995]. The smallest simulated earthquake whose source occupies one element has a vertical size equal to the thickness of the upper crust and should have a magnitude $M \geq 6$. For given loading and material parameters of the model, this lower cutoff defines an appropriate element size. The isometric cell choice for the simulation, instead of a narrow and elongated one, eliminates any preferred direction related to the numerical procedure. The amount of energy released during a seismic event is proportional to stress drop multiplied by the volume of the elements involved in the rupture process. This energy scales linearly with the width of the damage zone. Similarly, the seismic moment of a simulated earthquake and the associated plastic strain (see below) also increase proportionally to the width. The rate of elastic energy accumulation does not depend on the element size, but the energy release rate is proportional to it. Thus the change of element size leads to proportional change of the duration of the earthquake cycle for a model with a single fault. However, this effect is much weaker than that induced by small changes of the critical strain diagonality (see Appendix A), and it may be compensated by a small variation of ξ_0 . The focus of this work is on different possible dynamic regimes of evolving seismicity patterns for a major strike-slip

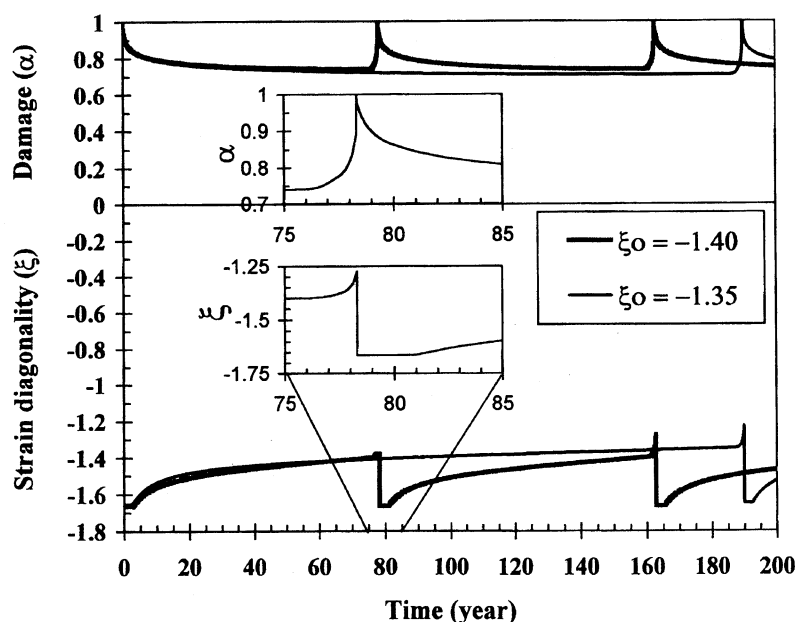


Figure 5. Strain diagonality (ξ) and damage (α) evolution during earthquake cycle for a single fault model. Insert shows ξ and α variations before and after a seismic event. With increase of the critical strain diagonality (ξ_0) from -1.40 to -1.35 the length of the cycle increases about twice. Damage rate constants are $C_d=0.1$ s⁻¹; $C_f=10^{-10}$ s⁻¹; $C_2=0.05$.

fault system. Since we use the trade-off between fault zone width and ξ_0 to get a realistic average repeat time of large earthquakes, we expect the results on these issues for earthquakes in the approximate magnitude range $6.0 \leq M \leq 8.5$ not to be affected significantly by the grid size choice.

The interseismic period ends and the preseismic period begins when the current strain diagonality achieves a critical value for “subcritical” crack growth. The condition for damage onset ($\xi > \xi_0$) is satisfied and the damage begins to increase. Increasing damage leads to higher strain and further damage localization since the elastic moduli explicitly depend on the damage variable α as discussed by *Lyakhovsky et al.* [1997a]. In addition, for our elastic upper crust governed by damage rheology, the temporal derivative of stress in equations (13) and (A4) includes a term of a type $\partial\sigma/\partial\alpha \cdot d\alpha/dt$ that implicitly decreases the effective material stiffness for increasing damage. The process of material weakening and increasing preseismic deformation is localized in a small region (one numerical element), and for a grid size that represents realistic fault zone width (e.g., order 100 m or less) this “nucleation” process is not expected to produce an observable surface geodetic signal. The preseismic period ends when the localized damage reaches a critical value for brittle failure. The duration of this period depends on a damage rate coefficient C_d . This is set here as $C_d = 0.3\text{--}0.5 \text{ s}^{-1}$, a value which is somewhat below those ($0.5\text{--}5 \text{ s}^{-1}$) estimated from laboratory data [*Lyakhovsky et al.*, 1997a], to increase the stability of the numerical procedure. This slightly enlarges the duration of the nucleation weakening period that ends (Figure 5) with initiation of dynamic rupture when α achieves its critical value ($\alpha = \alpha_{cr}$). At the present modeling level we do not simulate details of the dynamic failure process and use a quasi-static solution. However, we introduce a quasi-dynamic procedure for rupture front propagation by recalculating the stress field after a stress drop in every numerical element and by incorporating a dynamic weakening of material everywhere via reduction of the critical value of the damage parameter to $\alpha_{dynamic}$ given by

$$\alpha_{dynamic} = \alpha_{cr} - \sqrt{\tau_r \frac{d\alpha}{dt}} \quad (15)$$

where τ_r is relaxation time given by the ratio of the effective viscosity of the damaged crust to its rigidity. The effective viscosity is moderated by the rate of damage accumulation and depends on the model structure in a manner similar to that described above. For the numerical simulations shown below we vary τ_r between 10^{-2} and 1 year. Detailed derivation and implications of (15) are discussed by A. Agnon et al. (manuscript in preparation). For infinite viscosity the dynamic weakening (15) implies an ideal brittle behavior. In the initial set of calculations for a single fault model the parameter τ_r is set unrealistically big, so every nucleation of a brittle failure event in some part of the damage zone propagates through the entire area. This leads to relatively simple failure histories consisting of system-sized events occurring in a single fault zone. In section 3.2 we use smaller values of τ_r , allowing for the arrest of the rupture front in regions where the preexisting damage is not high enough. The role of this parameter will be explored further in section 3.2 dealing with distributed faulting.

As mentioned earlier, the brittle failure process is set here to drop initially the deviatoric stresses in the rupture zone to zero and to conserve only the volumetric stress. This stress

drop produces coseismic displacements in the surrounding material and nonreversible plastic strain in the rupture zone. The latter is modeled in simpler traditional frameworks as slip on a planar fault surface. The amount of plastic strain in the damage zone is related to the total amount of slip deficit during an earthquake cycle. The magnitude of the simulated earthquake may be estimated from the known stress drop or the accumulated plastic strain. Following *Madariaga* [1979], the tensor of the seismic moment (M_{ij}) is a volumetric integral of the stress drop

$$M_{ij} = \int_V \Delta\sigma_{ij} dv \quad (16a)$$

where V is the volume of the zone sustaining the stress drop ($\Delta\sigma_{ij}$). This relation is more general than the usual definition of the seismic moment

$$M_{ij} = \int_V C_{ijkl} \Delta\epsilon_{kl}^{(plastic)} dv \quad (16b)$$

where C_{ijkl} is the tensor of elastic moduli. Unfortunately, stress drop determinations are subjected to large uncertainties, and it is not possible in practice to obtain $\Delta\sigma_{ij}$ through the failure zone. Another option is to calculate the potency [*Ben-Menahem and Singh*, 1981] of the event as

$$P_{ij} = \int_V \Delta\epsilon_{kl}^{(plastic)} dv \quad (17)$$

Heaton and Heaton [1989], *Ben-Zion* [1989] and *Amelung and King* [1997] argued that the potency provides a better physical measure of the overall size of an earthquake than the seismic moment. In general, routine processing of seismograms for seismic moment (the zero spectral asymptote) provides no information on material properties, and the rigidity part of the moment is assumed rather than being derived from the data. Furthermore, the inclusion of rigidity in the definition of the seismic moment makes it ambiguously defined for the general case of laterally heterogeneous fault zones. For simple cases of constant elastic properties the potency, also referred to as geometric moment [*Kanamori and Anderson*, 1975], is equal to the seismic moment divided by rigidity. However, for the more realistic case discussed here, this simple relation does not hold a priori since the elastic properties evolve very strongly during the earthquake rupture process. The relation between moment and potency in our model with variable rigidity is not unique, and we analyze it with results based on the distributed damage model of section 3.2. We calculate the earthquake magnitudes in two ways: (1) from the seismic moment (16a) using the empirical relation $M = 2/3 [\log_{10}(M_0) - 16.1]$, where $M_0 = (M_{ij} M_{ij})^{1/2}$ is moment in dyne cm [*Hanks and Kanamori*, 1979], and (2) from the potency using the empirical relation $M = (2/3) \log_{10}(P) + 3.6$, where $P = (P_{ij} P_{ij})^{1/2}$ is in $\text{km}^2 \text{ cm}$ [*Ben-Zion and Rice*, 1993].

Figure 6 shows a good correlation between event magnitudes in the model with distributed damage (see section 3.2) calculated in these two different ways. Note that we used the moment definition (16a) that does not include explicitly material properties, rather than the relation usually used by seismologists to derive seismic moment from data, (16b) or its Fourier transform. The relation (16b) contains material properties and does not have a unique well-defined meaning for deformation process involving spatio temporal changes of elastic moduli studied here and presumably occurring during earthquakes.

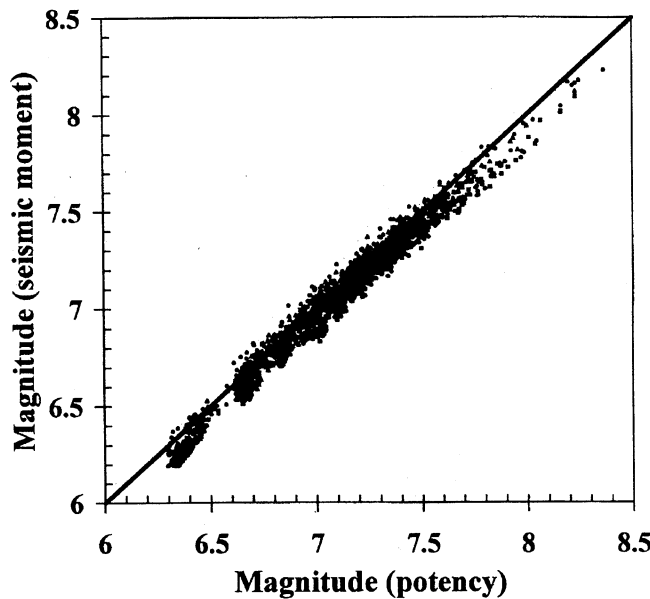


Figure 6. Correlation between earthquake magnitudes calculated using seismic moment and potency based on model runs with different parameters.

In contrast to rupture nucleation, material healing after the seismic event occurs in the whole rupture zone simultaneously during the postseismic period. The same term $\partial\sigma/\partial\alpha \cdot d\alpha/dt$ that was responsible for stiffness reduction during the preseismic period reverses its sign and locks the damage zone by increasing the effective rigidity. This produces rapid transient strengthening that is followed by actual material healing (i.e., decrease of damage). This temporarily prevents an increase of the strain diagonality during a few years after the event (Figure 5) when the damage level is still very high. The characteristic time scale for material healing depends on values of the coefficients C_1 , C_2 in the equation for healing in the damage rheology model [Lyakhovsky *et al.*, 1997a, equation 42]. Figure 7 shows the decrease of damage due to confining pressure during 3 years of the postseismic period for three different pairs (C_1 , C_2) used in the simulations and referred to as fast, medium and slow. The material recovers 20–30% of its damage-free rigidity to $\alpha=0.7$ – 0.8 during the first half a year for all the cases. After that, the healing rate significantly decreases, and total recovery after 3 years of the postseismic period approaches 30–50% ($\alpha=0.5$ – 0.7). The most active period of locking of the damage zone is around half a year. This time scale is comparable with that reported by Savage and Svarc [1997] based on postseismic deformation associated with the 1992 Landers earthquakes and by Zhao *et al.* [1997] based on inferred states of stress before and after the 1994 Northridge earthquake. The rate and level of material recovery do not significantly change the behavior of a single fault model, but they govern the style of strain localization and earthquake statistics in the model with distributed faulting discussed in section 3.2.

3.2. Regional Earthquakes and Fault Networks

The following simulations are aimed at the study of the process of damage-strain localization, evolution of fault

networks, and statistics of simulated seismic events. We start each simulation with randomly distributed damage (and hence stiffness) such that the stress and strain are in equilibrium. Instead of the localized step function velocity of the upper mantle, employed in section 3.1, we use a linear velocity profile at the base of the substrate. The values of the base velocities on the left and right edges are equal to those used for the single-fault model. All material properties except those responsible for the kinetics of the damage process are the same as in the single-fault model.

The ratio between the rate of healing and the rate of loading is central to the model behavior. It controls the style of localization and type of earthquake statistics. We define the loading timescale, τ_L , as the time needed to reaccumulate (rebuild) the elastic shear strain from a stress drop of a brittle failure to a level that gives strain ratio ξ equal to the critical value ($\xi_0=1.35$). This loading timescale depends on the relaxation time of the viscoelastic system of upper and lower crust layer (Figure 2), and the amplitude of the imposed mantle motion. The healing time scale, τ_h , characterizes the rock memory, or how long a material that is broken in a seismic event stays significantly weaker than its unbroken surrounding. This value depends on confining pressure and healing rate coefficients (Figure 7). In the following set of simulations we keep the loading timescale constant and vary the healing rate coefficients.

The first set of simulations corresponds to slow healing (solid line in Figure 7). The geometry of the high damage zone formed by the first few events reflects the initial random distribution and keeps its overall geometrical features during more than a thousand model years. Most of the seismic events occur in the same zone and occupy a similar volume. The slip accumulates in a relatively narrow and regular damage zone (Figure 8 and Plate 1 on the left) and preserves some features of the initial random damage for a long period. This behavior leads to a preferred event size manifested as a narrow local maximum in noncumulative frequency-size statistics of model earthquakes. We refer here to such statistics with a preferred event size as characteristic

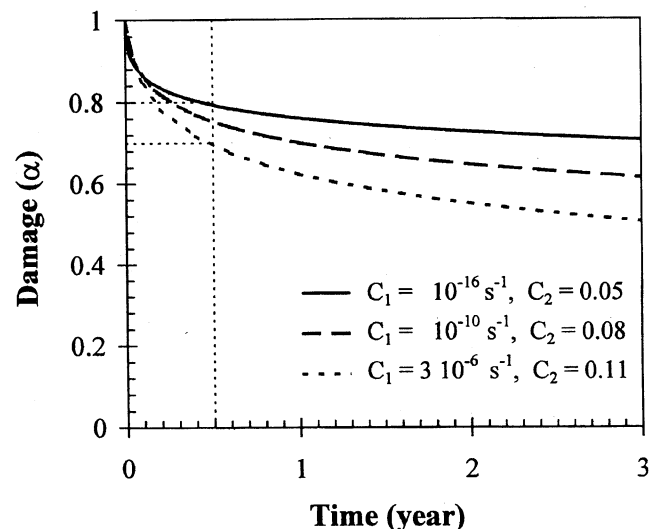


Figure 7. Material recovering (damage versus time) under constant lithostatic pressure and zero shear stress. Three different sets for healing rate constants represent slow, medium, and fast healing.

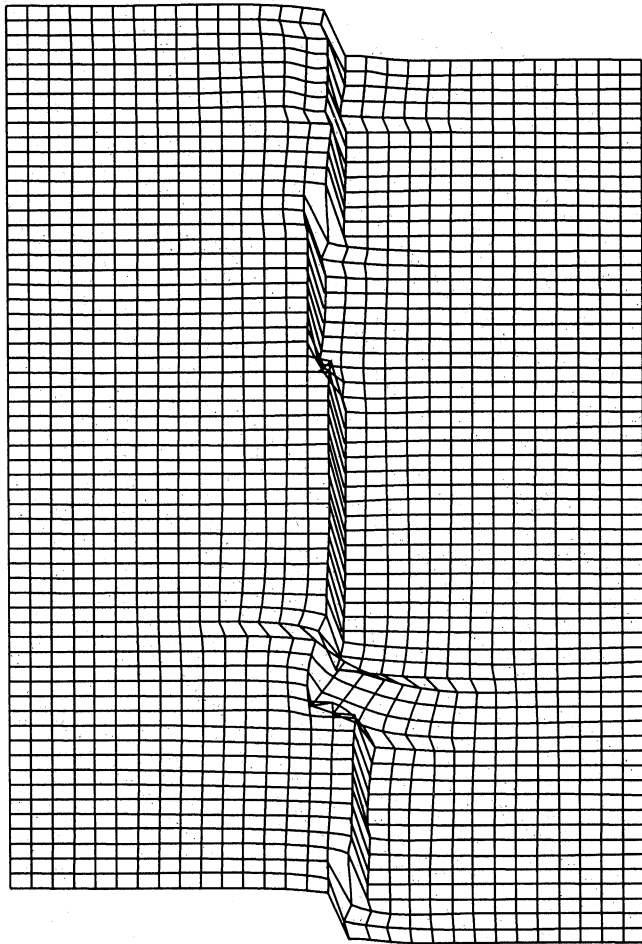


Figure 8. Typical distribution of the cumulative strain for the model with slow healing. The slip and high damaged material (Plate 1 on the left) are concentrated in relatively narrow and regular zone.

earthquake distribution. The magnitude of the characteristic events strongly depends on the prescribed dynamic weakening. Relatively small values ($\tau_r=0.1$ year) give a maximum in the non-cumulative statistics for earthquakes with magnitude from 6.7 to 6.9 (Figure 9). Relatively large values of the dynamic weakening ($\tau_r=0.3$ year) shift the maximum to event magnitude of ~ 7.2 -7.4 without changing the form of the statistics.

Increasing of the material recovering (dashed line in Figure 7) leads to broadening of the maximum in the non-cumulative frequency-size statistics (Figure 10), changing it towards a Gutenberg-Richter power law distribution. The cumulative number of events with magnitude larger 7.2 could be approximated by a linear relation with $b=2.5$, a value significantly larger than the observed b value near unity. Further small increasing of the material recovering (dotted line in Figure 7) significantly increases the geometrical complexity of the evolving damage zones (Figure 11 and Plate 1 on the right). The large conjugate structures accumulate significant amount of displacement. The offsets between fault segments increase, and some of them produce branching that prohibits their further propagation. Part of the simulated region can preserve distributed damage (Plate 1) without

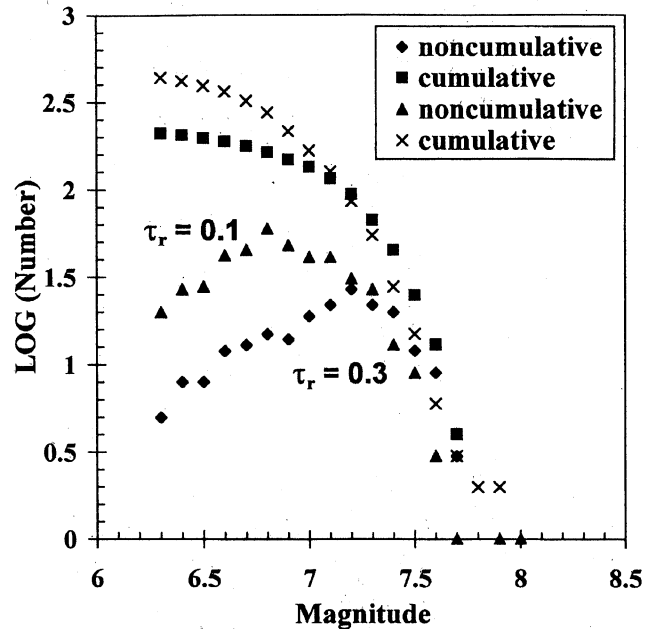


Figure 9. Cumulative and noncumulative frequency-size statistics of seismic events for slow damage healing (long material memory) and different τ_r . The statistics have a preferred event size manifested as a narrow local maximum in the noncumulative distribution.

being involved in the process of damage localization for a long time. The frequency-size earthquake statistics for the fast healing case are much closer to the Gutenberg-Richter distribution. A linear relation with $b=1.3$ can approximate the

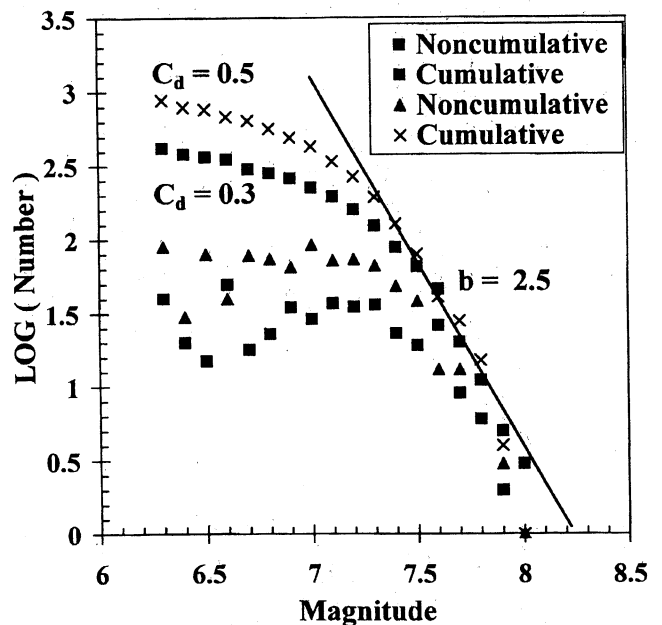


Figure 10. Frequency-size statistics of seismic events for medium damage healing and different damage rate constants C_d . The cumulative number of large events may be approximated by a linear relation with $b=2.5$, a value significantly larger than typically observed b values near unity.

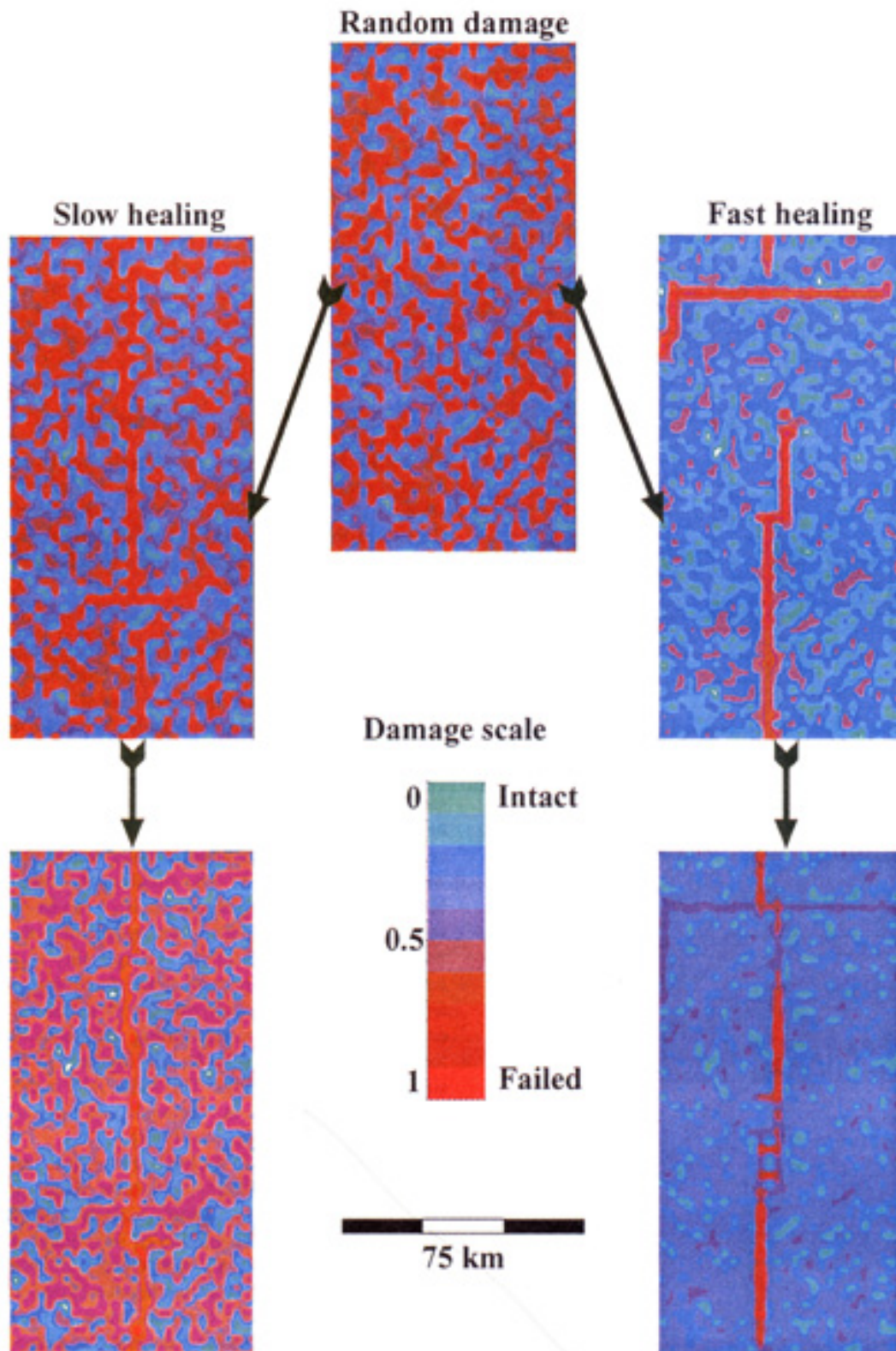


Plate 1. Typical patterns of damage distribution and its evolution for (left) a model with slow healing, and (right) a model with fast healing.

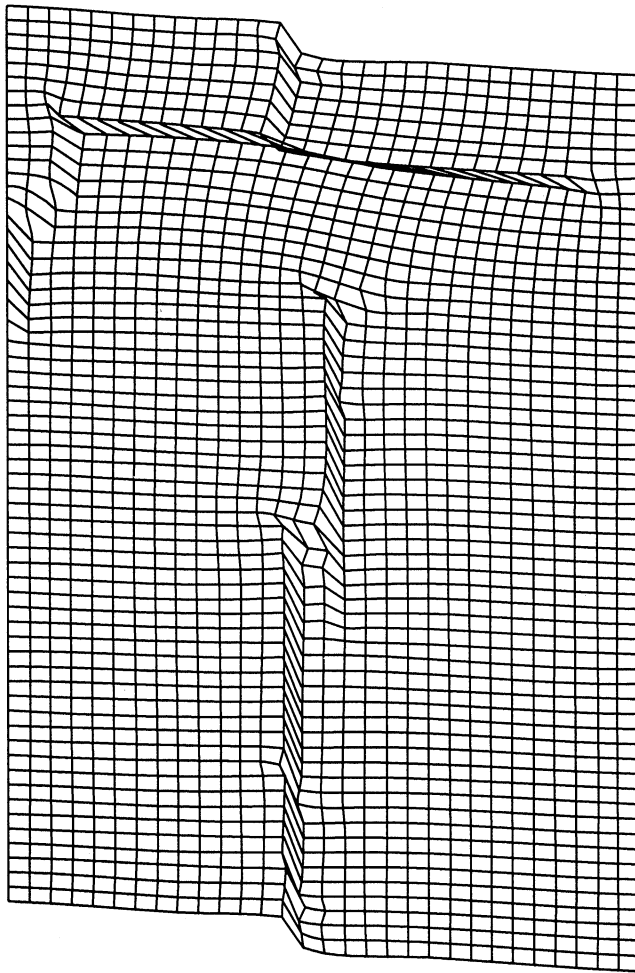


Figure 11. Cumulative strain for a model with fast healing. Large conjugate structures accommodate significant amount of displacement.

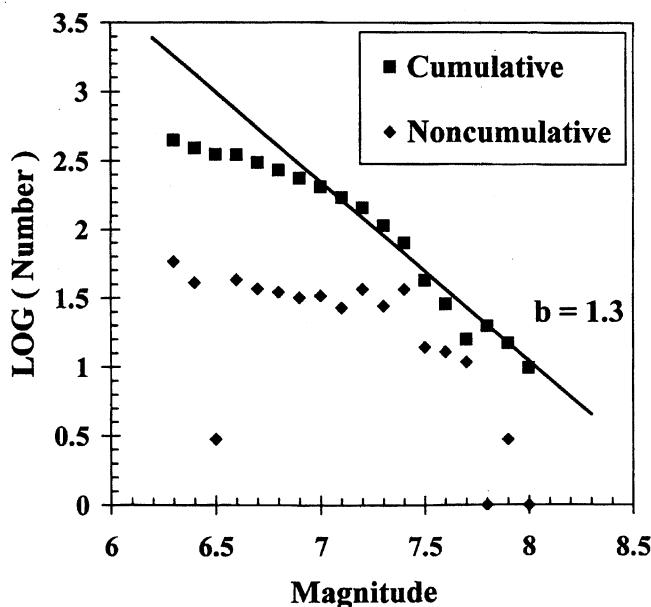


Figure 12. Frequency-size statistics of seismic events for fast damage healing. A linear relation with $b=1.3$ can approximate the cumulative statistics of events with magnitude larger 6.8.

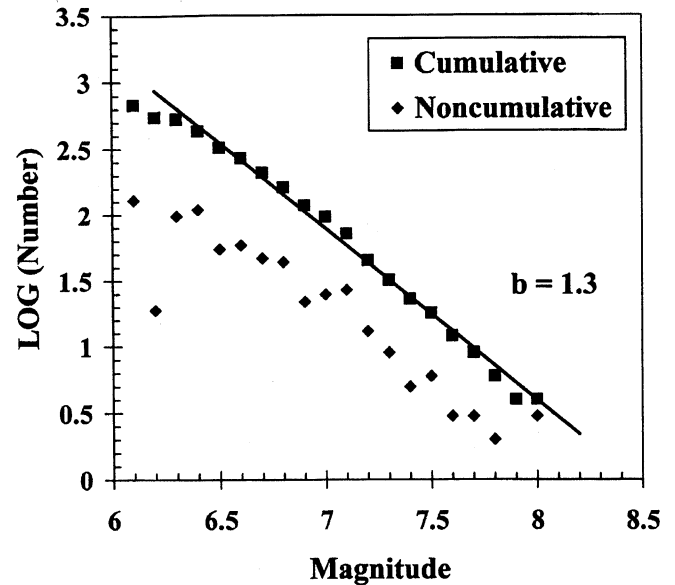


Figure 13. Frequency-size statistics of seismic events for fast damage healing and small spatial variations of material strength. The statistics are compatible with the Gutenberg-Richter power law distribution for the entire simulated magnitude range (compare with Figure 12).

cumulative statistics of events with magnitude larger 6.8 (Figure 12). If the crust properties other than damage are perfectly homogeneous, the geometrical complexity of the fault system (Figure 11 and Plate 1 on the right) is not preserved during a period of many earthquake cycles. In this case a broad damage zone has a tendency to collapse into a narrow one similar to that shown in Figure 8 and Plate 1 on the left, sustaining characteristic earthquakes that produce deviation from the Gutenberg-Richter distribution for events with magnitude 7.0-7.3. The situation is, however, different for a slightly heterogeneous crust. Even small spatial variations in material parameters representing the strength of the upper crust as critical strain diagonality (ξ_0) or dynamic weakening (τ_r) or both prevent the foregoing regularization tendency and lead to fault zones with sustained geometrical complexities. The corresponding frequency-size statistics follow a power law with $b=1.3$ (Figure 13) for the entire simulated magnitude range. This value falls in the range of the observed $b=0.7-1.35$ in regional and global earthquake catalogs [Frohlich and Davis, 1993].

In such cases we also find that the seismic response of the model (Figure 14) exhibits long-term fluctuations that we have called “mode switching” activity [Ben-Zion *et al.*, 1999b]. The temporal evolution of seismicity switches back and forth between intervals with clusters of intense seismic activity lasting several large earthquake cycles and relatively quiet intervals of similar duration. Most earthquakes in the relatively active intervals have magnitudes above 6.8, and the frequency-size statistics in these portions of the response are compatible with the characteristic earthquake distribution. During the relatively quiet periods the event magnitudes are below $M=6.8$, and the frequency-size statistics are compatible with a truncated power law. The earthquake sequences before and after the largest model events (Figure 14, insets) show diverse patterns. The largest earthquake in the left inset is surrounded by a cluster of strong events, while the largest

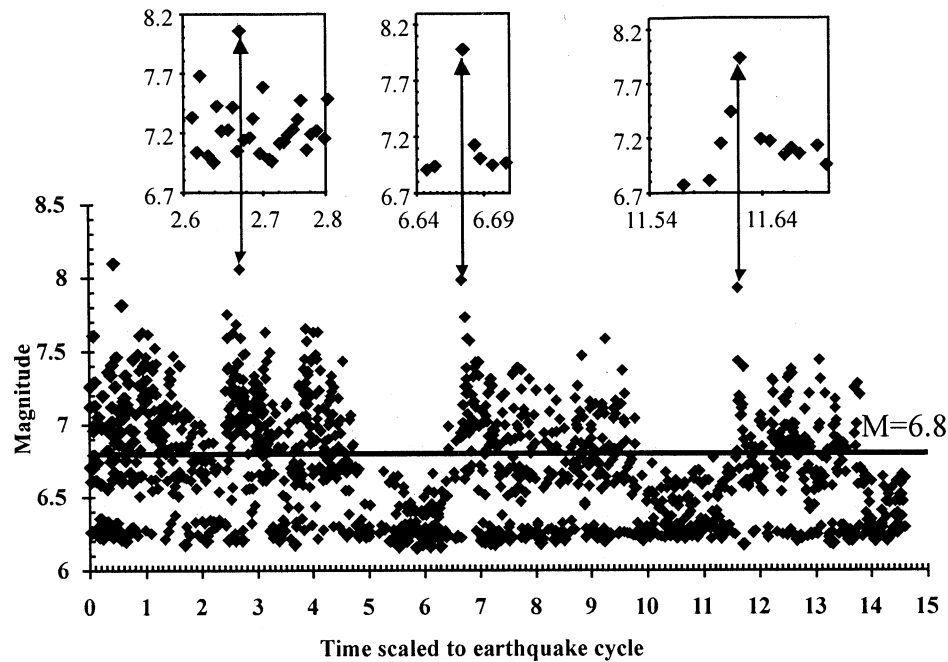


Figure 14. Seismicity record with mode-switching activity, consisting of time intervals of several large earthquake cycles with clusters of large events, separated by intervals of similar duration with only small and intermediate size earthquakes. The insets show a variety of different event sequences before and after the largest model earthquakes.

events in the middle and right insets have narrow and wide foreshock and aftershock sequences, respectively. If the material recovery is increased further, so that effective healing occurs in a small fraction of a large earthquake cycle, the behavior changes from mode switching to sustained response with disordered networks of faults and frequency-size statistics compatible with the Gutenberg-Richter distribution [Ben-Zion *et al.*, 1999b].

4. Discussion

We introduce a nonlinear continuum mechanics framework for simulating the coupled evolution of earthquakes and faults in a regional model consisting of a seismogenic upper crust governed by damage rheology over a layered viscoelastic half-space. The damage rheology [Lyakhovsky *et al.*, 1997a] incorporates nonlinear and irreversible aspects of rock deformation, including localization and nucleation phases, strength evolution with slip and time, and branching from the main rupture plane. The model accounts for interactions between evolving fault zones, and interaction of the brittle seismogenic crust with a viscoelastic substrate. The results illustrate the importance of four different timescales for the evolution of the crust and frequency-size earthquake statistics.

Our previous analysis [Lyakhovsky *et al.*, 1997a] focused on three of the controlling timescales. One for degradation, τ_d , is the inverse of the modulus degradation rate and characterizes the duration of the period between damage onset and brittle failure. The second one for loading, τ_l , is the inverse of the total strain rate and characterizes the time needed to reaccumulate the elastic strain from a stress drop of a brittle failure to the critical level for onset of damage. The duration of the earthquake cycle in a single fault model is equal to the sum of τ_d and τ_l . The third timescale, τ_r , comes from the Maxwell relaxation of elastic stresses and is equal to

the viscosity divided by rigidity. The dynamic weakening (15) depends on the value of τ_r and thus controls the preferred event size for the characteristic frequency-size distribution. The present work highlights the role of an additional timescale, the healing time τ_h , controlling fault pattern evolution and earthquake statistics in the model for the crust-mantle system [see also Ben-Zion *et al.*, 1999b].

Lyakhovsky *et al.* [1997a] show a stick-slip cycle with three stages: degradation, instability, and healing. These stages correspond to the interseismic, coseismic, and postseismic stages of a seismic cycle, respectively. The existence of a fourth preseismic stage is indicated by the analysis of localization in a degradable and purely elastic one-dimensional material [Lyakhovsky *et al.*, 1997a]. The localization is also anticipated in a degradable viscoelastic medium if the loading is faster than the viscous relaxation of elastic stress in the undamaged medium ($\tau_r > \tau_l$). A. Agnon *et al.* (manuscript in preparation) further show that once the degradation is faster than the relaxation ($\tau_r > \tau_d$), the preseismic equivalent stage of subcritical crack growth is terminated and a dynamic stress drop ensues. These results are similar to a transition from quasi-static nucleation phase to dynamic rupture in rate- and state-dependent [e.g., Okubo, 1989; Dieterich, 1992; Ben-Zion and Rice, 1997] and slip-weakening [Shibazaki and Matsuura, 1992; Ohnaka, 1996] frictional frameworks.

The full four stages in the seismic cycle are manifested in the present model. The model provides an improvement over the generalized Elsasser model [Li and Rice, 1987] in simulating geodetic signals with algebraic decay compatible with observations, close to the fault as well as in the far field. The calculated velocity profile for the fault parallel velocity fits well an arctangent curve (Figure 4) that represents the average geodetic measurements for the middle part of the earthquake cycle, all the way to the fault zone. During the

preseismic stage the elastic degradation is localized in a small area (one numerical element) and does not produce a significant geodetic signal. The dynamic weakening (15) and the proper 3-D elastic stress transfer calculated using the Green function (14) allow a failure in a hypocenter to extend into a region with possible complex geometry. The high healing rate in the postseismic period increases the effective rigidity across the fault zone. The corresponding healing time-scale, τ_h , is comparable to that reported by *Savage and Svarc* [1997] based on postseismic deformation following the 1992 Landers earthquakes and by *Zhao et al.* [1997] based on inferred states of stress before and after the 1994 Northridge earthquake.

Ben-Zion and Rice [1993, 1995] and *Ben-Zion* [1996] simulated seismicity patterns along a strike-slip fault with fixed prescribed heterogeneities. Their models represent approximately geometric disorder of a narrow fault zone by various types of disorder in strength properties. The simulations of the present work allow us to compare results from evolving complex structures with true geometric disorder to the simple planar approximations of *Ben-Zion and Rice*. In our model, the ratio between the time-scales of healing and loading (τ_h/τ_L) provides a guideline for understanding different dynamic regimes of fault pattern evolution and earthquake statistics that are evident in the simulations (Figures 8-14 and Plate 1). When the system has long-term memory (τ_h/τ_L is relatively high), it produces fast damage localization during a time interval less than one large earthquake cycle. This leads to the development of geometrically regular fault systems and frequency-size event statistics compatible with the characteristic earthquake distribution. In such cases, the event statistics are similar to those simulated by model realizations of *Ben-Zion and Rice* [1993, 1995] and *Ben-Zion* [1996] with relatively regular heterogeneities. Conversely, if the memory of the system is short-term (τ_h/τ_L is relatively low), it develops highly disordered fault systems and produces power law frequency-size statistics compatible with the Gutenberg-Richter distribution. In these cases, the event statistics are similar to those simulated by model realizations of *Ben-Zion and Rice* with highly disordered heterogeneities. The correlation between fault complexity and frequency-size statistics suggested by the present model and the previous simulations of *Ben-Zion and Rice* [1993, 1995] and *Ben-Zion* [1996] is in agreement with field and seismological observations of *Wesnowsky* [1994] and *Stirling et al.* [1996].

For intermediate ratio of timescales τ_h/τ_L , the results exhibit switching of response between two distinct modes of activity (Figure 14). The first mode is associated with clusters of large earthquakes and frequency-size statistics compatible with the characteristic earthquake distribution, while the second mode is characterized by relatively low moment/potency release and frequency-size event statistics following a truncated power law. The average duration of each activity mode scales with the time interval of a large earthquake cycle in the system. As discussed by *Ben-Zion et al.* [1999b] and *Dahmen et al.* [1998], similar activity switching exists for a range of parameters in the simpler planar models of *Ben-Zion and Rice* [1993, 1995]. Long-term switching of seismic activity between relatively active and relatively quiet time intervals has been indicated by disturbed sediments and surface faulting in the Dead Sea transform, in a continuous record spanning 50 thousand years [*Marco et al.*,

1996]. Qualitatively similar alternating deformation phases have been documented in the eastern California Shear Zone [*Rockwell et al.*, 2000], the Great Basin Province in the western United States [*Wallace*, 1987], the Loreto basin, Baja California [*Dorsey et al.*, 1997], and other locations summarized by *Ben-Zion et al.* [1999b]. In the Dead Sea transform sites studied by *Marco et al.* [1996], each active period may exceed 10,000 years and contain up to 15 large events, sevenfold to tenfold more than in the relatively quiet periods. A similar picture would emerge from Figure 14 for events with $M > 7.5$. The first half of the record shows a cluster with 17 $M > 7.5$ events, whereas the second shows only 4 events. The scaling of this model to the Dead Sea transform should account for an order of magnitude lower loading rate [*Joffe and Garfunkel*, 1987; *Ellenblum et al.*, 1998; *Pe'eri et al.*, 1999].

Our simulated event statistics depend in general on the space-time window of the observational domain; that is, temporal sequences from different regions are not statistically equivalent among themselves, nor are they statistically equivalent to spatial ensembles of earthquakes occurring in different zones. The results imply that extrapolations of observed earthquake statistics based on low-magnitude seismicity and short time intervals to large earthquake behavior over longer duration require careful justification tailored to the particular case in hand.

Appendix A: Damage Mechanics

The macroscopic effects of distributed cracking and other types of damage require treatment by constitutive models that include nonlinear stress-strain relations together with material degradation and recovery. In order to simulate a process of fracturing in terms of continuum mechanics, a nondimensional intensive damage variable α is introduced. The variable α is related to the density of microcracks in a laboratory specimen, or the density of small faults in a crustal domain. For the sensitivity of elasticity to distributed cracks, consider the response of a single crack. Across the crack there is no cohesion, so under extension the crack dilates, diminishing the resistance to loading. Conversely, under normal pressure, contact forces across the crack resist deformation. *Lyakhovsky et al.* [1997b] derive the macroscopic stress-strain relations for a 3-D elastic solid accounting for the cumulative effect of microcrack opening and closure and test the solution against rockmechanics experiments. Following their formulation the elastic potential is written as

$$U = \frac{1}{\rho} \left(\frac{\lambda}{2} I_1^2 + \mu I_2 - \gamma I_1 \sqrt{I_2} \right) \quad (A1)$$

where λ and μ are Lamé constants, $I_1 = \epsilon_{kk}$ and $I_2 = \epsilon_{ij}\epsilon_{ij}$ are two independent invariants of the strain tensor ϵ_{ij} , and γ is an additional elastic modulus (summation notation is assumed). The second order term with the new modulus γ accounts for microcrack opening and closure in a damaged material. The term incorporates nonlinear elasticity even for an infinitesimal strain, and it simulates abrupt change in the elastic properties when the loading reverses from compression to tension. The stress tensor σ_{ij} derived from (A1) is

$$\sigma_{ij} = \left(\lambda - \gamma \frac{\sqrt{I_2}}{I_1} \right) I_1 \delta_{ij} + \left(2\mu - \gamma \frac{I_1}{\sqrt{I_2}} \right) \epsilon_{ij} \quad (A2)$$

The stress-strain relation (A2) can be rewritten to mimic the usual form of Hook's law by introducing effective elastic moduli

$$\lambda^e = \lambda - \frac{\gamma}{\xi}, \quad \mu^e = \mu - \frac{1}{2} \gamma \xi \quad (\text{A3})$$

where the strain diagonality $\xi = I_1/\sqrt{I_2}$ characterizes the type of deformation. Agnon and Lyakhovsky [1995] suggested that the elastic moduli μ and γ are linear functions of the damage α and chose only the modulus λ to be constant:

$$\lambda = \lambda_0 = \text{const}; \quad \mu = \mu_0 + \alpha \xi_0 \gamma_r; \quad \gamma = \alpha \gamma_r \quad (\text{A4})$$

where γ_r is calculated from the conditions [Lyakhovsky et al., 1997a, equations (14) and (15)] of convexity loss for the maximum value of the damage variable ($\alpha=1$). They also related the critical strain diagonality (ξ_0) to the friction angle ϕ by considering the critical shear stress for Mohr-Coulomb sliding:

$$\tau = \tan(\phi) \sigma_n \quad (\text{A5})$$

where σ_n is normal stress. Consider a saw-cut interface between two intact blocks in a friction experiment carried out under confining pressure. The condition for fault slip is then [Agnon and Lyakhovsky, 1995]

$$\xi_0 = \frac{-\sqrt{3}}{\sqrt{2q^2(\lambda_0/\mu_0 + 2/3)^2 + 1}} \quad (\text{A6})$$

where $q = \sin(\phi)/[1 - \sin(\phi)/3]$. Physically, (A6) means that the critical strain diagonality ξ_0 is some modification of the internal friction. Savage et al. [1996] draw a connection between macroscopic friction measured on saw-cut specimens and internal friction that characterizes shear fracture of intact rock. They write the strength of an intact rock as the sum over the plane of the incipient fault of both friction on closed microcracks and strength of the remaining grains. The approach taken by damage mechanics [Lyakhovsky et al., 1997a] extends that connection. Under confining pressures sufficient for closure of microcracks, stress concentration may arise only once the shear stress meets the frictional criterion. Then favorably oriented cracks slide and load their tips giving rise to damage increase. The difference between the frictional strength of prefaulted surfaces and the strength of the intact rock is given by the excess stress that is needed to increase the damage from its initial value to critical. That stress difference is rate-dependent, and it can be calculated readily from the model damage kinetic equations:

$$\frac{d\alpha}{dt} = C \cdot I_2 (\xi - \xi_0), \quad (\text{A7})$$

where C depends on ξ and α (see Lyakhovsky et al. [1997a] for thermodynamic derivations and more details).

Appendix B: Numerical Scheme

The system of equations that should be solved numerically consists of the equation of motion of the hybrid model and equations describing damage evolution and transient stress redistribution. Here we rewrite all these equations with brief comments on the numerical approach to their solution. We start with the equation of motion (13)

$$H \iint_S G_{kn} \frac{\partial}{\partial t} \frac{\partial \sigma_{nm}}{\partial x_m} ds + \frac{Hh}{\eta} \frac{\partial \sigma_{km}}{\partial x_m} = \frac{\partial u_k}{\partial t} - V_{plate}^{(k)} \quad (\text{B1})$$

which is solved by iterative procedure similar to "FLAC" [Cundall and Board, 1988]. For the known elastic strain and all material parameters from the previous time step an iterative procedure gives new velocities ($V_i = \partial u_i / \partial t$) in the simulated area. The computational Lagrangian mesh consists of quadrilateral elements, which are subdivided into pairs of constant strain triangles, with different diagonals. This double overlay scheme ensures symmetry of the solution by averaging results obtained on two meshes. Linear triangular element shape functions L_k ($k=1,3$) are defined as

$$L_k = a_k + x b_k + y c_k \quad (\text{B2})$$

where a_k , b_k , and c_k are constants and (x, y) are grid coordinates. These shape functions are used to interpolate the nodal velocities $v_i^{(k)}$ within each triangle element: defined as

$$v_i(x, y) = \sum_{k=1}^3 v_i^{(k)} L_k \quad (\text{B3})$$

Equation (B3) enables calculation of the strain rate tensor e_{ij} in each triangle. The stress tensor σ_{ij} is calculated using elastic deformation, ε_{ij} , from the previous time step according to nonlinear stress-strain elastic relations [Lyakhovsky et al., 1997b]. Being partially differentiated respect to time coordinate, equation (A2) gives the relation between $\partial \sigma_{ij} / \partial t$ and strain rate tensor e_{ij} :

$$\begin{aligned} \frac{\partial \sigma_{11}}{\partial t} = & \lambda e_{11} - \gamma \frac{\varepsilon_{11} e_{11} + \varepsilon_{22} e_{22} + 2\varepsilon_{12} e_{12}}{\sqrt{I_2}} \\ & - \gamma \varepsilon_{11} \left[\frac{e_{11} + e_{22}}{\sqrt{I_2}} - \xi \frac{\varepsilon_{11} e_{11} + \varepsilon_{22} e_{22} + 2\varepsilon_{12} e_{12}}{I_2} \right] \\ & + (2\mu - \gamma \xi) e_{11} + \frac{d\lambda}{dt} I_1 - \frac{d\gamma}{dt} \sqrt{I_2} \\ & + \left(2 \frac{d\mu}{dt} - \frac{d\gamma}{dt} \xi \right) \varepsilon_{11} \end{aligned} \quad (\text{B4a})$$

$$\begin{aligned} \frac{\partial \sigma_{22}}{\partial t} = & \lambda e_{22} - \gamma \frac{\varepsilon_{11} e_{11} + \varepsilon_{22} e_{22} + 2\varepsilon_{12} e_{12}}{\sqrt{I_2}} \\ & - \gamma \varepsilon_{22} \left[\frac{e_{11} + e_{22}}{\sqrt{I_2}} - \xi \frac{\varepsilon_{11} e_{11} + \varepsilon_{22} e_{22} + 2\varepsilon_{12} e_{12}}{I_2} \right] \\ & + (2\mu - \gamma \xi) e_{22} + \frac{d\lambda}{dt} I_1 - \frac{d\gamma}{dt} \sqrt{I_2} \\ & + \left(2 \frac{d\mu}{dt} - \frac{d\gamma}{dt} \xi \right) \varepsilon_{22} \end{aligned} \quad (\text{B4b})$$

$$\begin{aligned} \frac{\partial \sigma_{12}}{\partial t} = & (2\mu - \gamma \xi) e_{12} \\ & - \gamma \varepsilon_{12} \left[\frac{e_{11} + e_{22}}{\sqrt{I_2}} - \xi \frac{\varepsilon_{11} e_{11} + \varepsilon_{22} e_{22} + 2\varepsilon_{12} e_{12}}{I_2} \right] \\ & + \left(2 \frac{d\mu}{dt} - \frac{d\gamma}{dt} \xi \right) \varepsilon_{12} \end{aligned} \quad (\text{B4c})$$

When stresses σ_{ij} and their temporal derivation $\partial \sigma_{ij} / \partial t$ are known, the summation on the left side of (B1) results in some vector (F_i) at each node, which is used for calculation of new velocities by simple iterative procedure:

$$v_i^+ = v_i^- + \chi (F_i + V_{plate} - v_i^-) \quad (\text{B5})$$

The iteration parameter χ provides a convergence of the procedure, which is repeated until the maximum difference between previous v_i^- and next v_i^+ values of velocities become negligibly small. Calculated velocity distribution is used for the Lagrangian transport equation for node coordinates:

$$x_i^+ = x_i^- + v_i dt \quad (B6)$$

and new total strain

$$\varepsilon_{ij}^+ = \varepsilon_{ij}^- + \frac{1}{2} \left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} \right) dt \quad (B7)$$

The new z component of elastic strain is defined according to the average lithostatic pressure ($\sigma_{zz} = -\rho g H/2$):

$$\varepsilon_{zz} = -\frac{1}{\lambda^e + 2\mu^e} \left[\lambda^e (\varepsilon_{xx} + \varepsilon_{yy}) + \rho g \frac{H}{2} \right] \quad (B8)$$

New thickness of the upper crust is calculated using continuity equation ($\text{div}(v_i)=0$) and local isostasy assumption:

$$H^+ = H^- \left[1 - \left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) dt \right] \quad (B9)$$

New damage distribution is calculated using evolution equation:

$$\alpha^+ = \alpha^- + [C_d I_2 (\xi - \xi_0)] dt \quad (B10a)$$

for degradation ($\xi > \xi_0$), and

$$\alpha^+ = \alpha^- + \left[C_1 \exp \left(\frac{\alpha}{C_2} \right) I_2 (\xi - \xi_0) \right] dt \quad (B10b)$$

for healing ($\xi < \xi_0$). The time step dt used for all these numerical procedures is selected after two conditions:

1. Damage change in each time step does not exceed some constant value ($\Delta\alpha_{\max}$) selected by comparing a series of similar runs

$$dt_{\text{damage}} = \frac{(\Delta\alpha)_{\max}}{\left(\frac{d\alpha}{dt} \right)_{\max}} \quad (B11)$$

2. Numerical stability of the parabolic equation (B1) based on the Fourier stability analysis requires that the time step Δt is less than half of the relaxation time of the shortest wave length, related to the grid size Δx [Ames, 1977]. The upper boundary for the time step used in the simulations is equal to one third of the characteristic time of the attenuation of the wave with length equal to Δx :

$$\Delta t_{\max} = \frac{1}{3} \frac{\Delta x^2}{hH\mu/\eta} < \frac{1}{3} \frac{\Delta x^2 + H\mu G_{yy} \left(\frac{2\pi}{\Delta x} \right)}{hH\mu/\eta} \quad (B12)$$

At some stage of evolution the level of damage in one or several elements achieves the critical level, corrected after the dynamic weakening (15). These elements can not further keep stress, which should be dropped. The immediate change of the stress $\Delta\sigma_{ij}$ is equivalent to distributed force along the element boundary, S , with vector n_k normal to it:

$$F_j = \Delta\sigma_{jk} n_k \quad (B13)$$

and the coseismic elastic displacements are calculated using the Green function (14):

$$u_k^{(e)} = \iint_S G_{kj} F_j ds \quad (B14)$$

The elements involved in the failure process accumulate plastic strain components that are recorded together with the values of the stress drop for calculation of the earthquake moment and potency.

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